# The Random Finite Element Method

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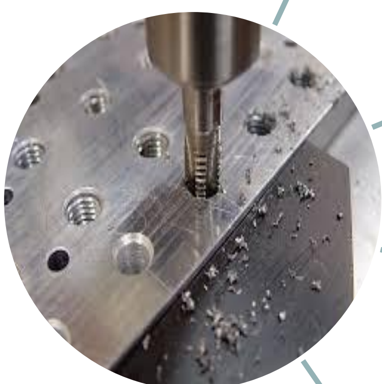
# Learning Objectives

## After this lecture, you will be able to:



**Source**
- List sources of uncertainty

**Models**
- Explain how uncertainty is incorporated in a simulation

**Control**
- Explain what is meant by irreducible uncertainty

**Effect**
- Explain how models are affected by variations in materials

**Control**
- Explain what is meant by reducible uncertainty

# Sources of Uncertainty

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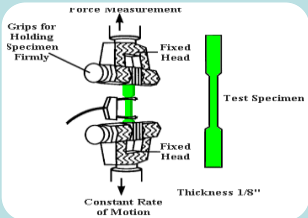
- Variability in input parameters
  - Inherent scatter in data
  - Lack of or insufficient input information
  - Modelling assumptions
  - Human error
- Includes
  - Loads
  - Material properties
  - Restraints
  - Geometry

Could be a combination of all of these!

# Irreducible Uncertainties

## Beyond our ability to completely control




### Material Properties

- Young's modulus different for samples of same part
- Due to variability in the internal composition

### Loads

- Randomness of environmental factors
- Wave height, wind velocity/direction

### Construction/manufacturing

- Site/factory conditions
- Quality of workmanship

# Reducible Uncertainties

## Within our ability to control

But, for some practical reason, such as time and cost, cannot be completely eliminated.

### Manufacturing

- When manufacturing a part containing a hole
- The diameter of the hole will vary from part to part
- This happens as the drill bit wears
- So there's uncertainty in geometry when modelling

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# Incorporating Uncertainty into Simulation

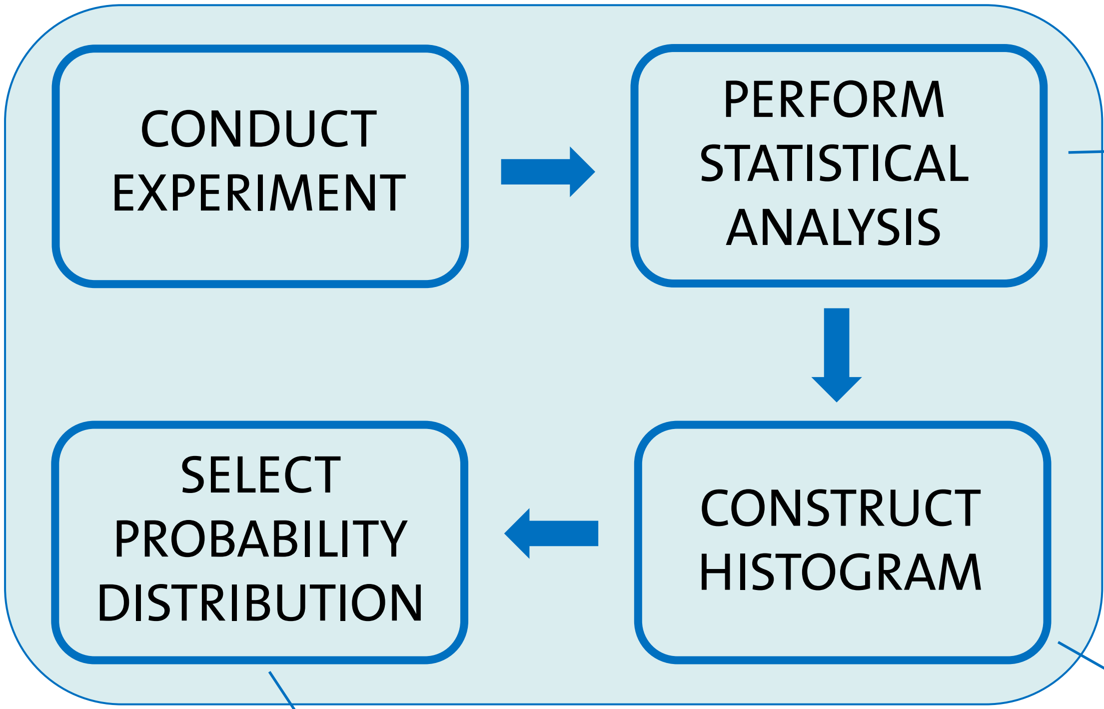
# Uncertainty Models

- If it is assumed that no variations occur in a parameter, we refer to the parameter as deterministic or single-valued.
  - Most finite element simulations assume all parameters are deterministic
  - We may carry out sensitivity analysis at the limits
- To incorporate uncertainties into a simulation, we need?

# Uncertainty Models

- If it is assumed that no variations occur in a parameter, we refer to the parameter as deterministic or single-valued.
  - Most finite element simulations assume all parameters are deterministic
  - We may carry out sensitivity analysis at the limits
- To incorporate uncertainties into a simulation, we need some type of uncertainty model.
- The most common approach is to use random variables
  - Relates the possible values to the probability of occurrence.

# Characterising Variability




Average, standard deviation, coefficient of variation (standard deviation/average)

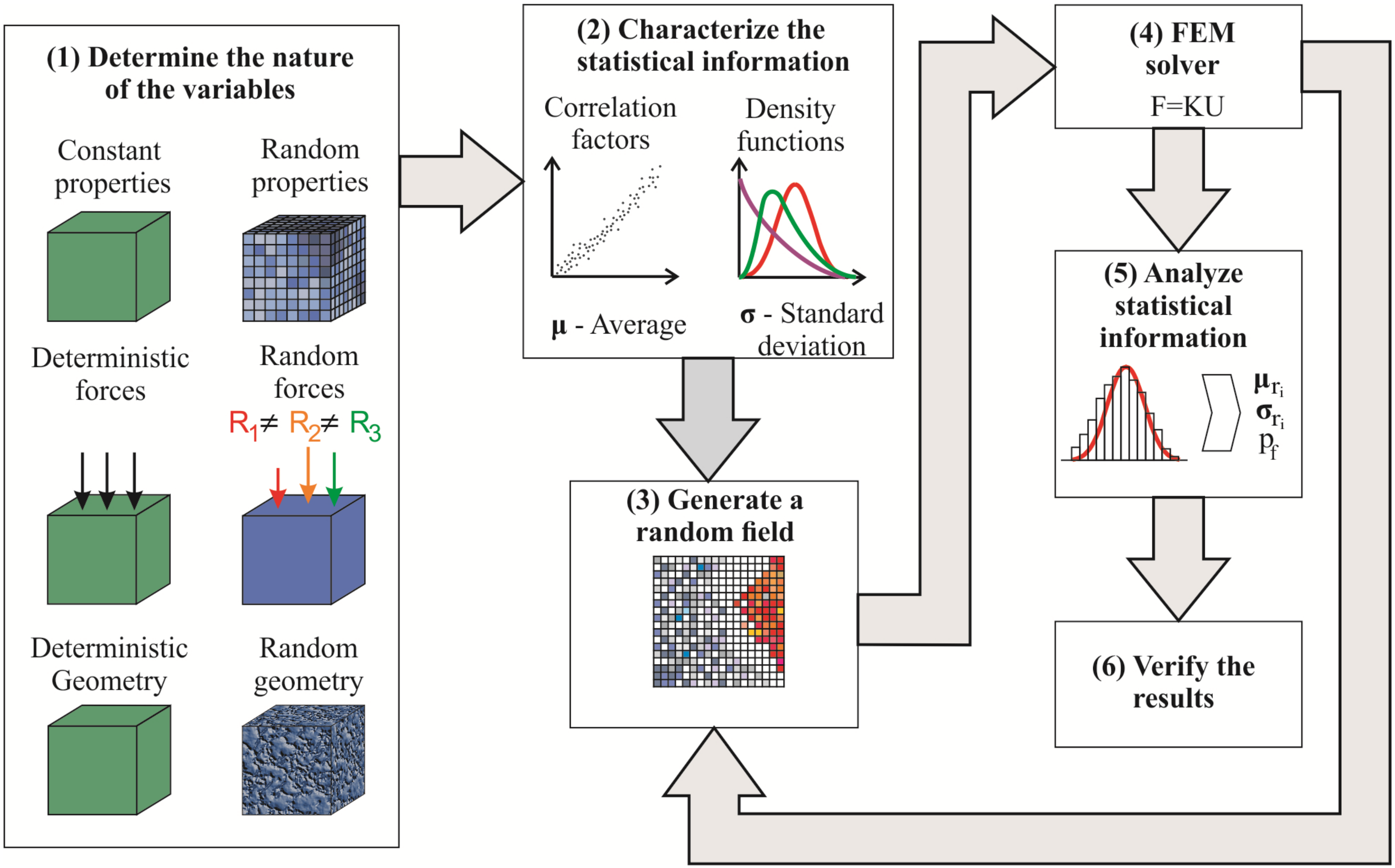
The raw distribution of the data for the random variable.

A simplified mathematical representation of the random variable.

# Analysis Methods

- How do we input uncertainties into a finite element model?
- Monte Carlo Simulation (MCS)
  - Take a random sample from the input distribution.
  - This sample produces one output sample of response.
  - The process is repeated a large number of times.
- Most Probable Point Method (MPP)
  - Uses optimisation algorithm to reduce number of analyses
- Hybrid
  - Combination of MPP and MCS to improve accuracy

# General Workflow



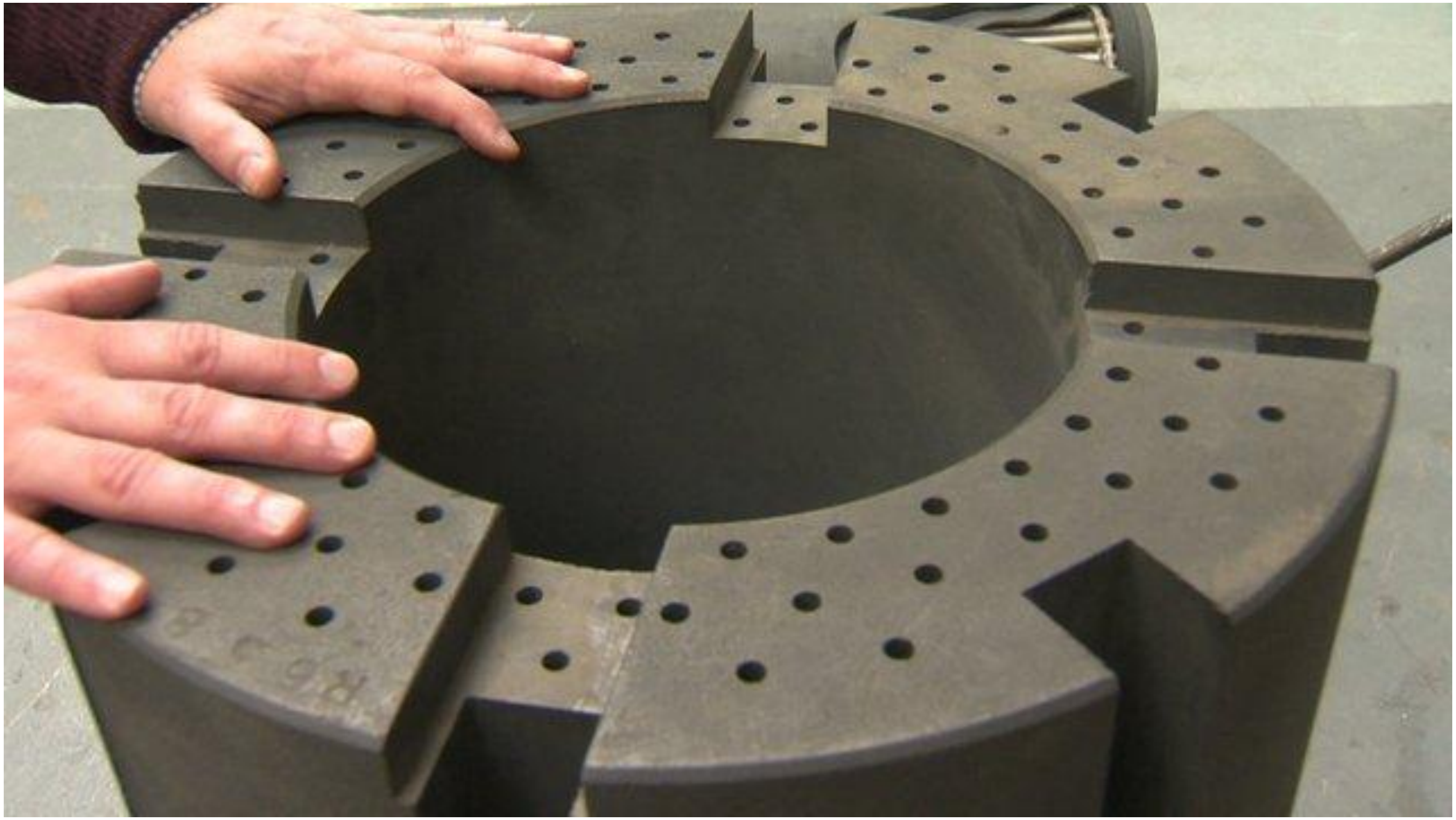
# Case Study
# Problem Definition

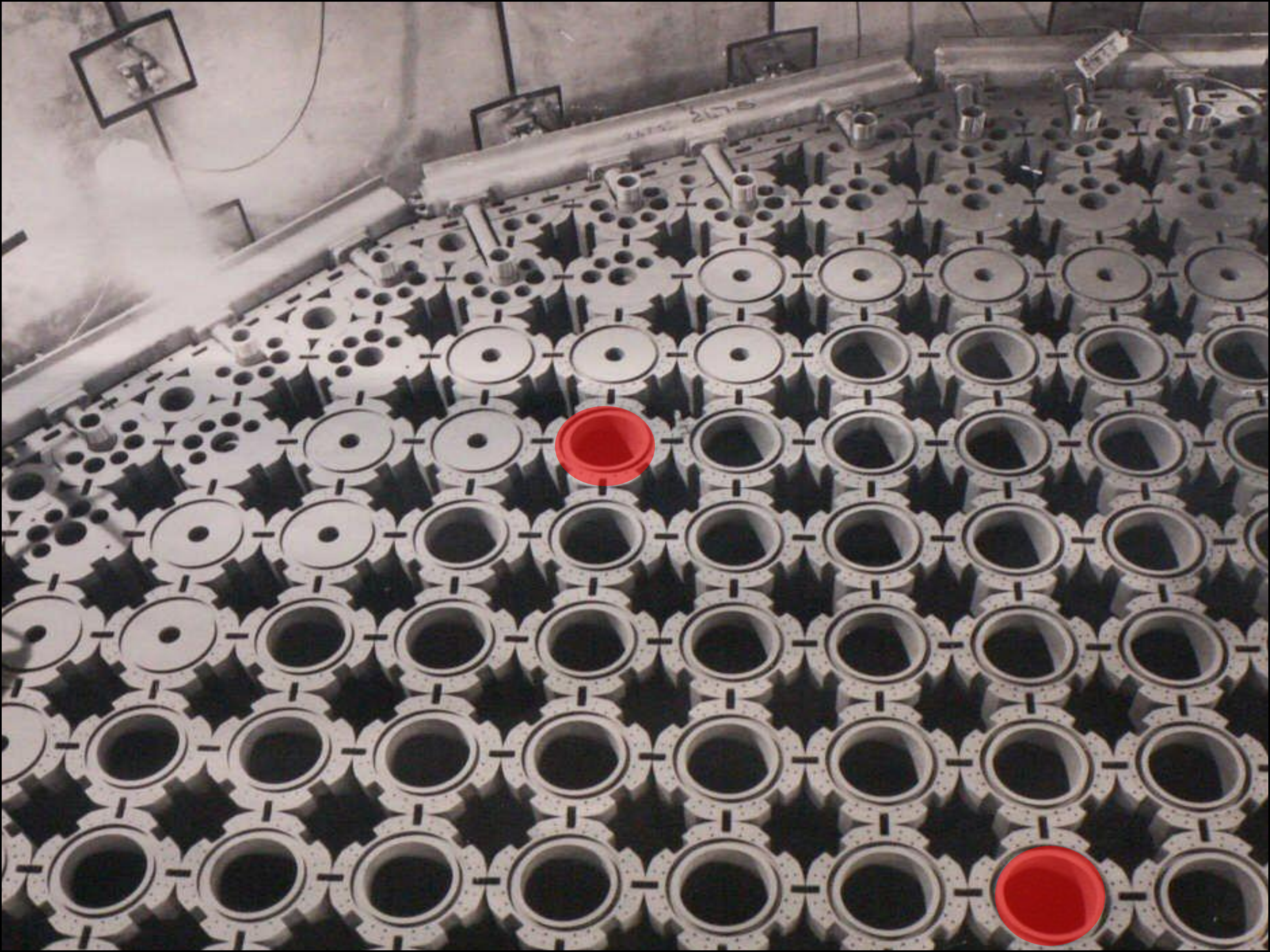
# Cracks found at reactor at Hunterston B nuclear power station

**By David Miller**
BBC Scotland environment correspondent

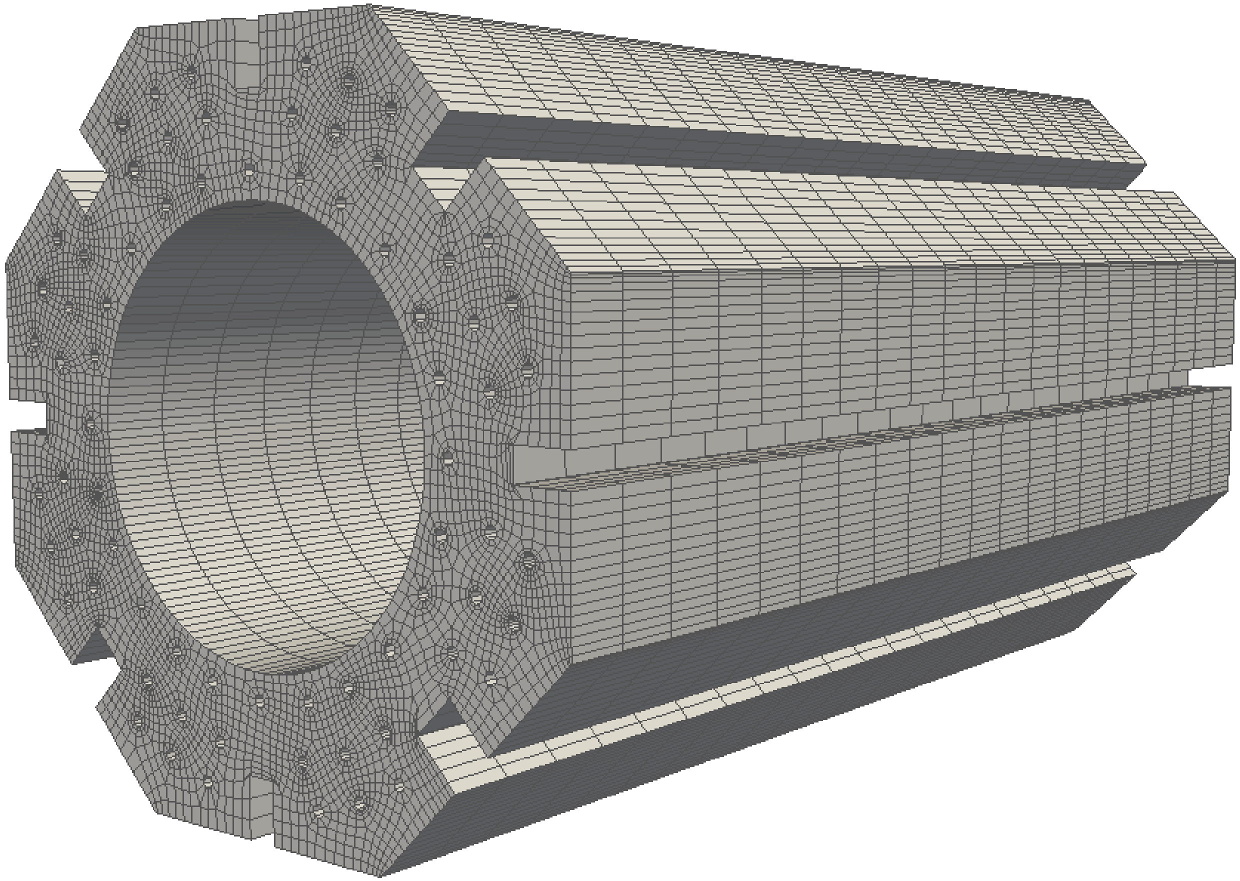
Source: http://www.bbc.com/news/uk-scotland-glasgow-west-29502329 on 6 October 2014

# Typical Nuclear Graphite Brick



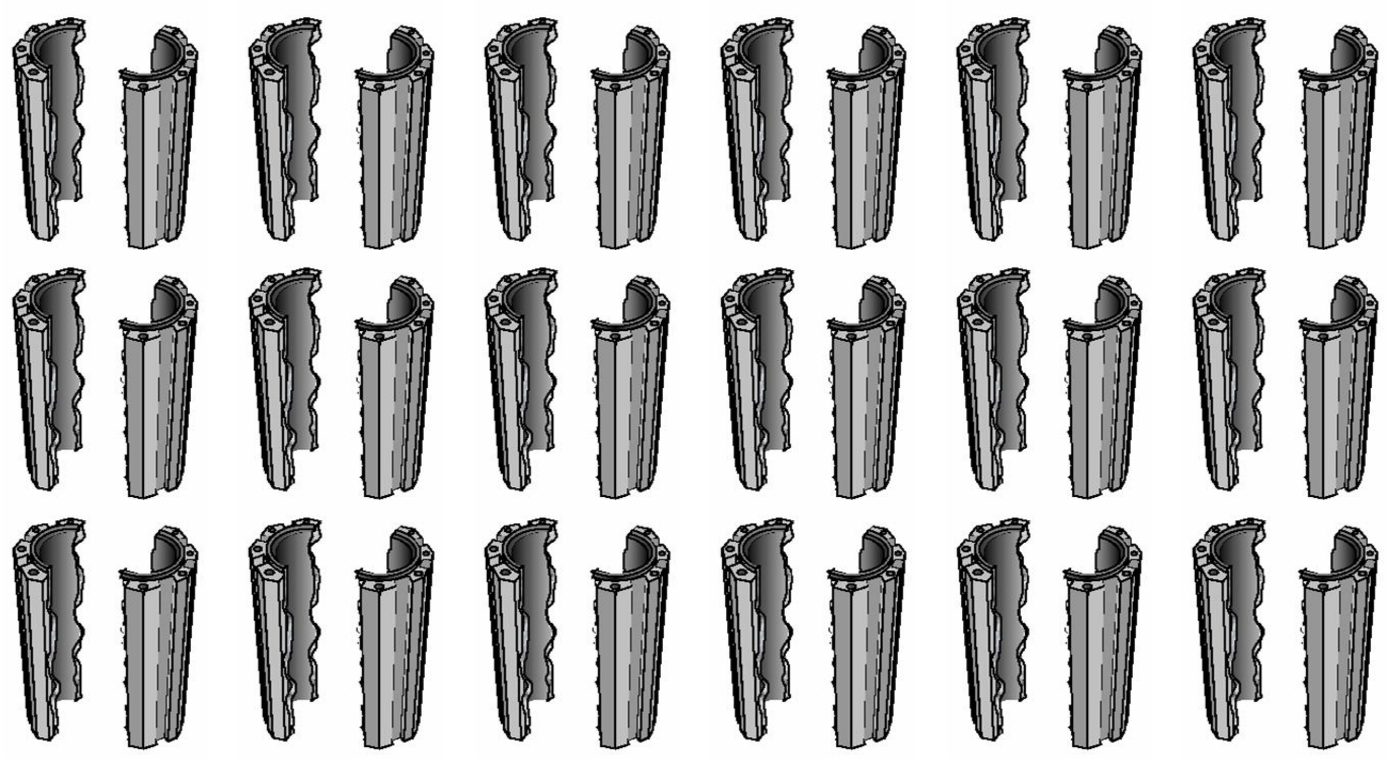


# Predictive Modelling



What is the consequence of a deterministic analysis?

# Not Quite Right



But despite very complicated constitutive models, the simulations aren't quite right. And if they were correct, they would predict that all the bricks would crack at the same time, which they don't!

# Assumption Used in Simulations

## Assertion

Expansion or contraction of the material, component or structure will NOT generate internal stresses as long as

- It is free to expand or contract (it is not fixed at any point)
- It rests on a smooth surface
- It is made of a uniform material with a single coefficient of thermal expansion.

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# Assumption Used in Simulations

## Assertion

~~Expansion or contraction of the material, component or structure will NOT generate internal stresses as long as~~

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# Hypothesis

## Hypothesis

Tiny spatial variations in material properties lead to pre-service stresses that influence the expected life of a brick

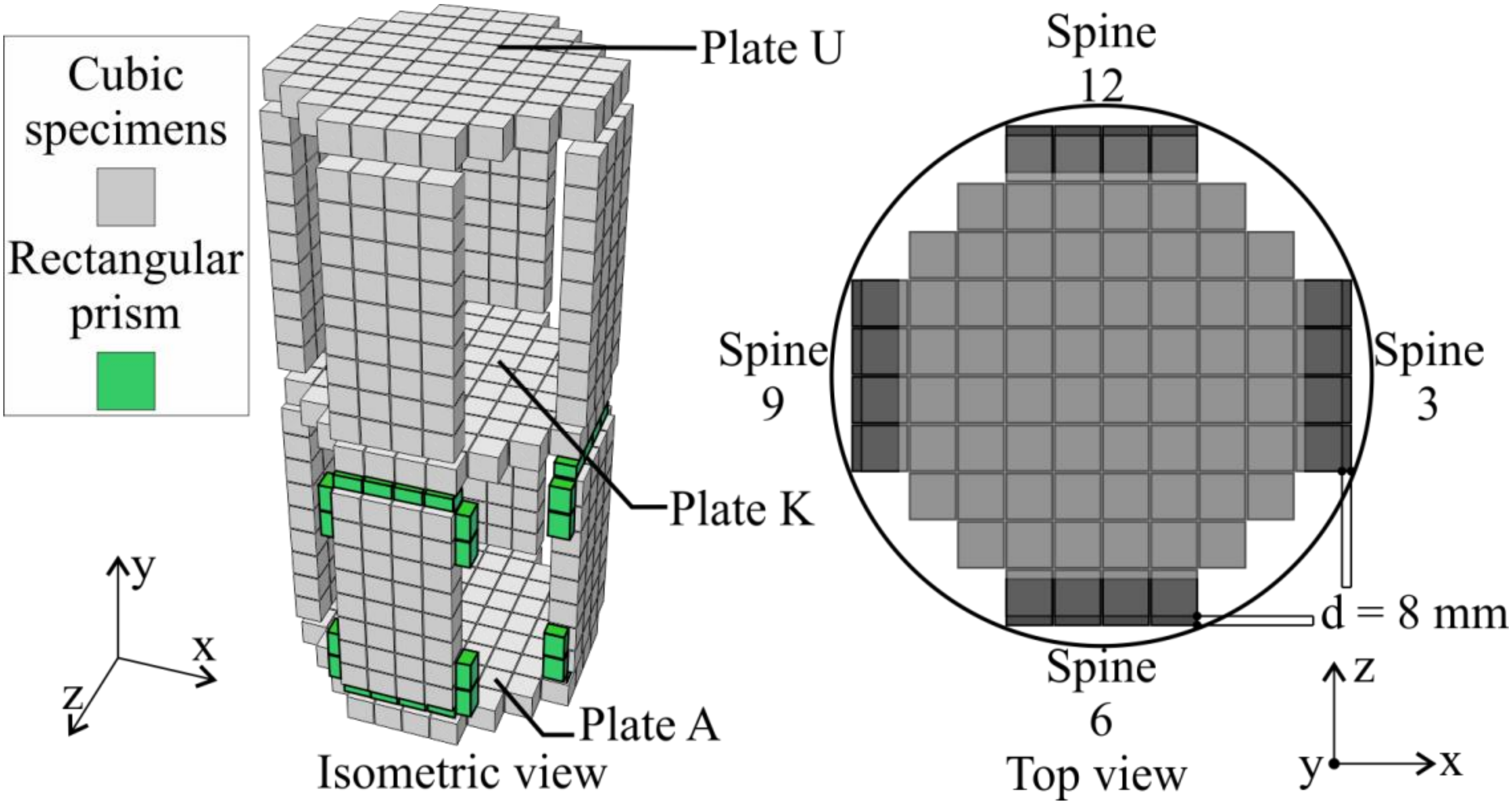
- It is free to expand or contract (it is not fixed at any point)
- It rests on a smooth surface
- It is NOT made of a uniform material with a single coefficient of thermal expansion.

# Experimental Work

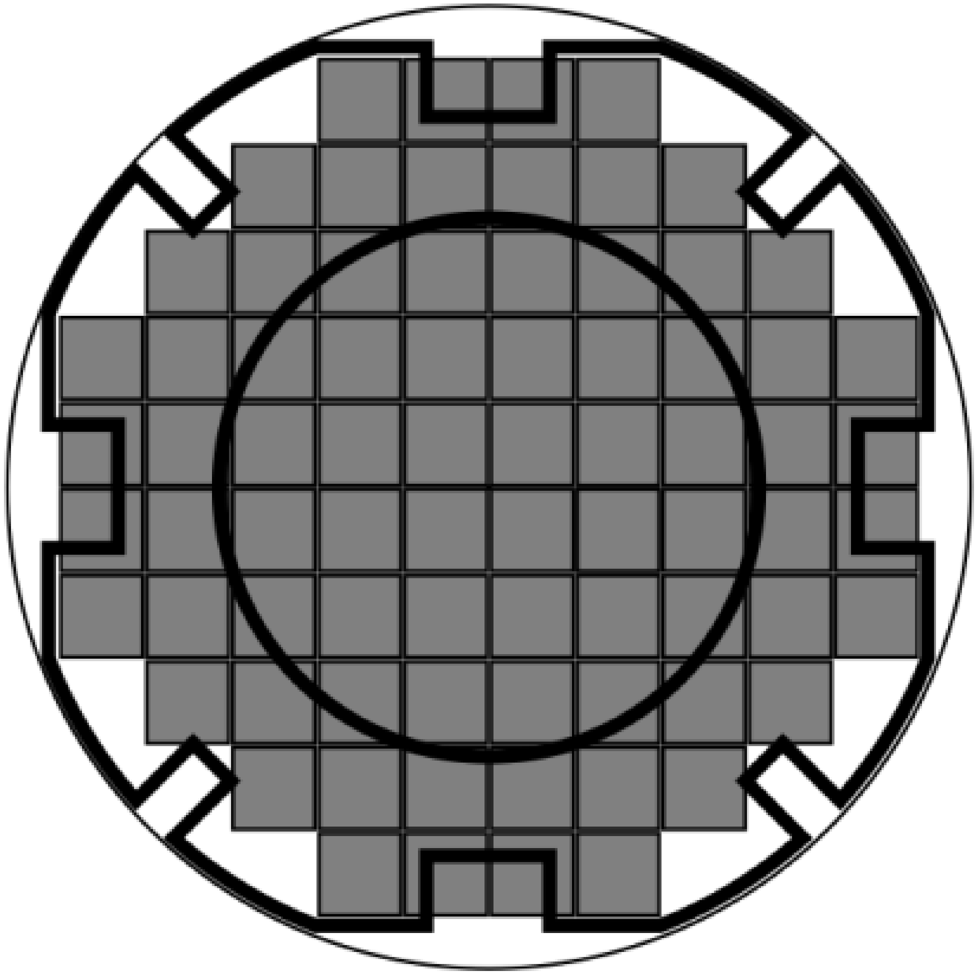
# Measurements

- Characterisation of the spatial variation in material properties – to determine values for the random variable
  - Density
  - Young's modulus
  - Poisson's ratio
- Coefficient of thermal expansion assumed to be linearly proportional to density.
  - Calibrated rather than measured
  - Temperature dependent

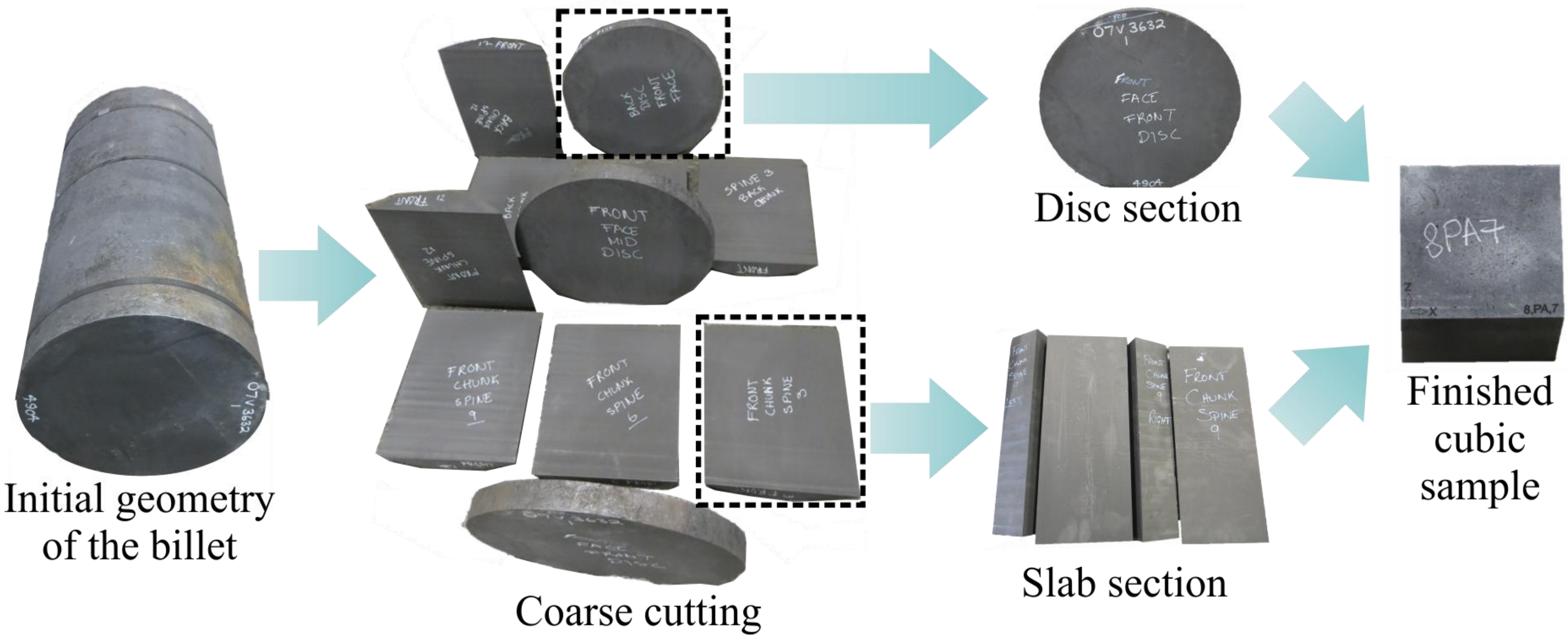
# Cutting Plan



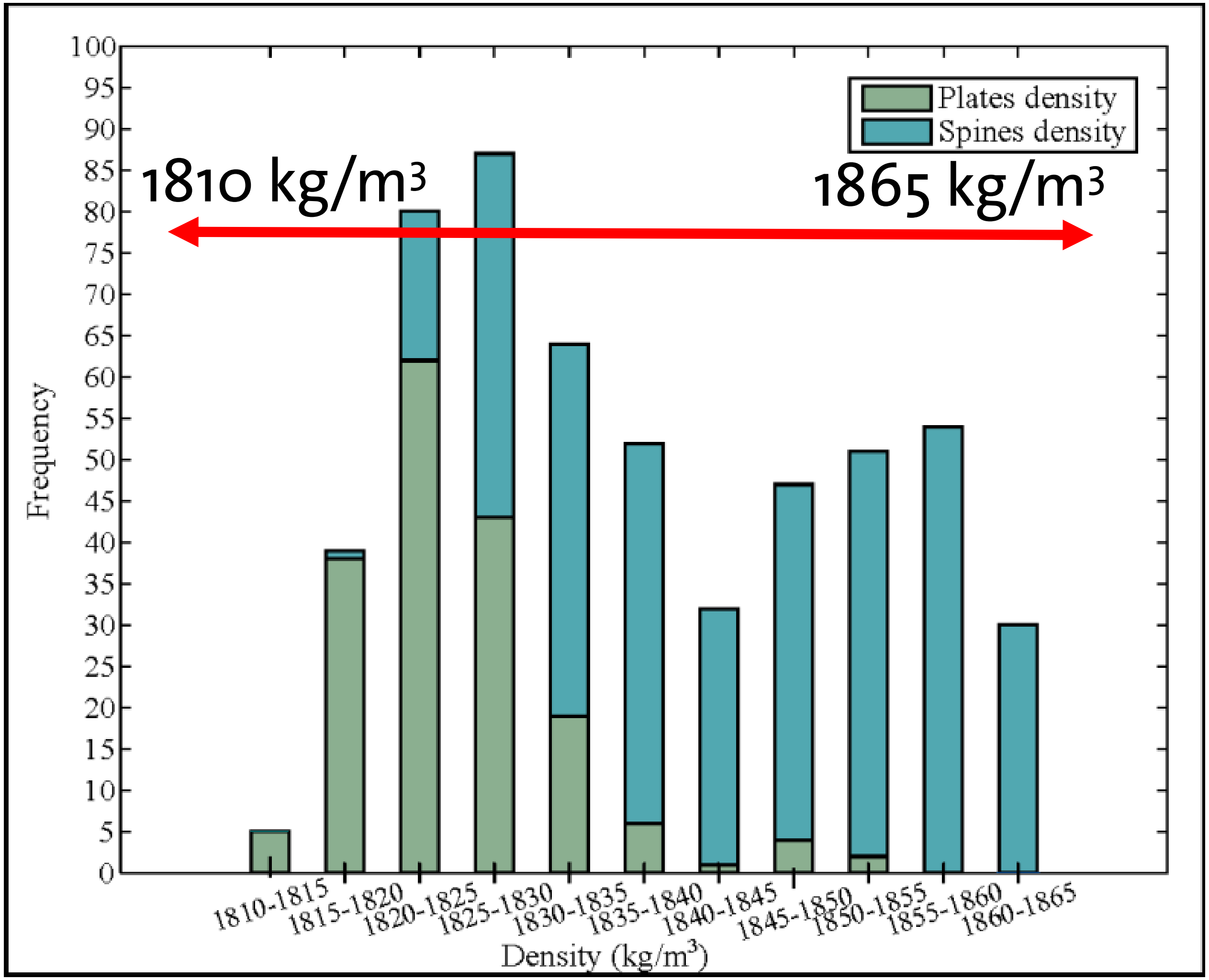
# Sampling Region



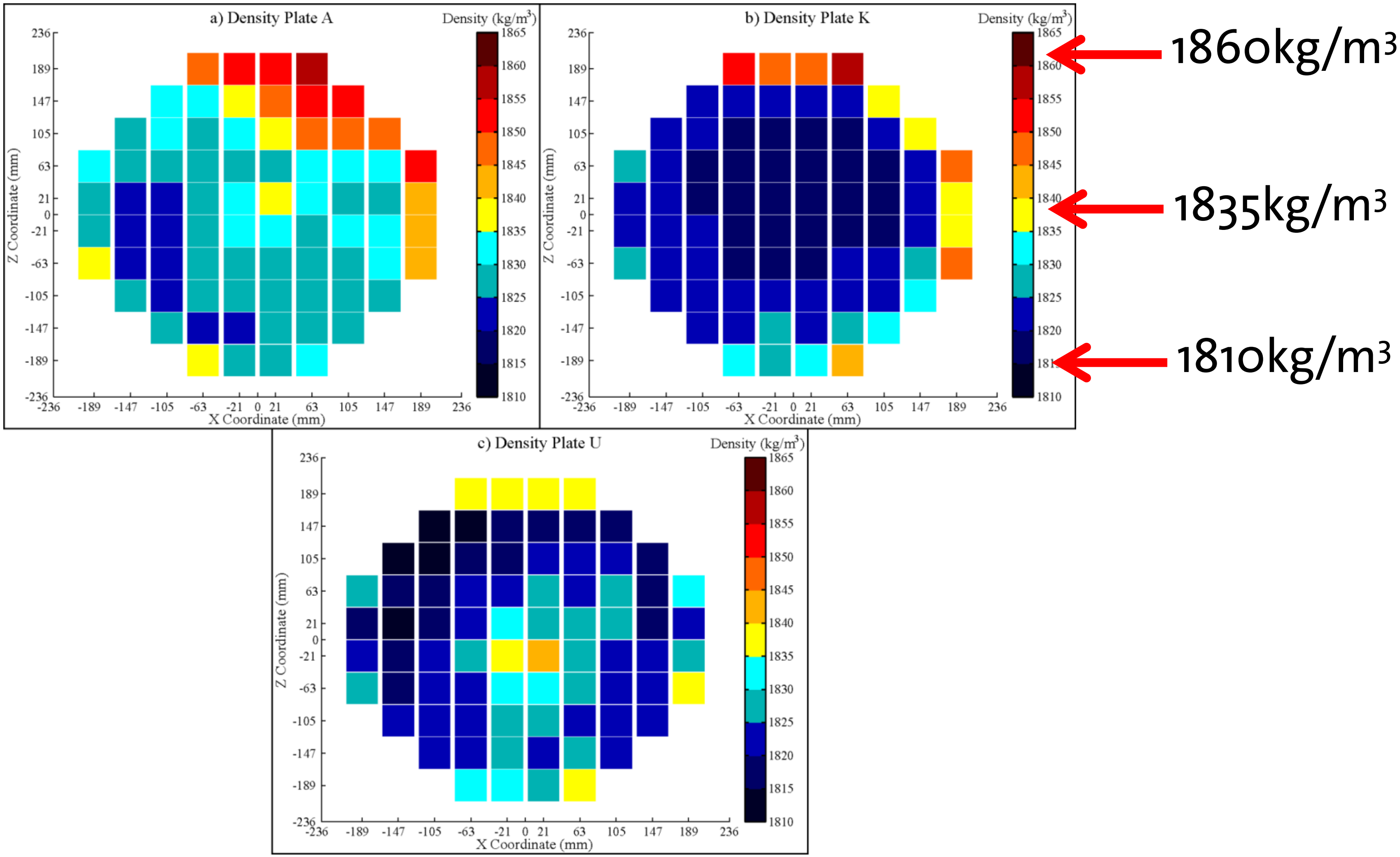
# Machining


Initial geometry of the billet
Coarse cutting
Disc section
Slab section
Finished cubic sample

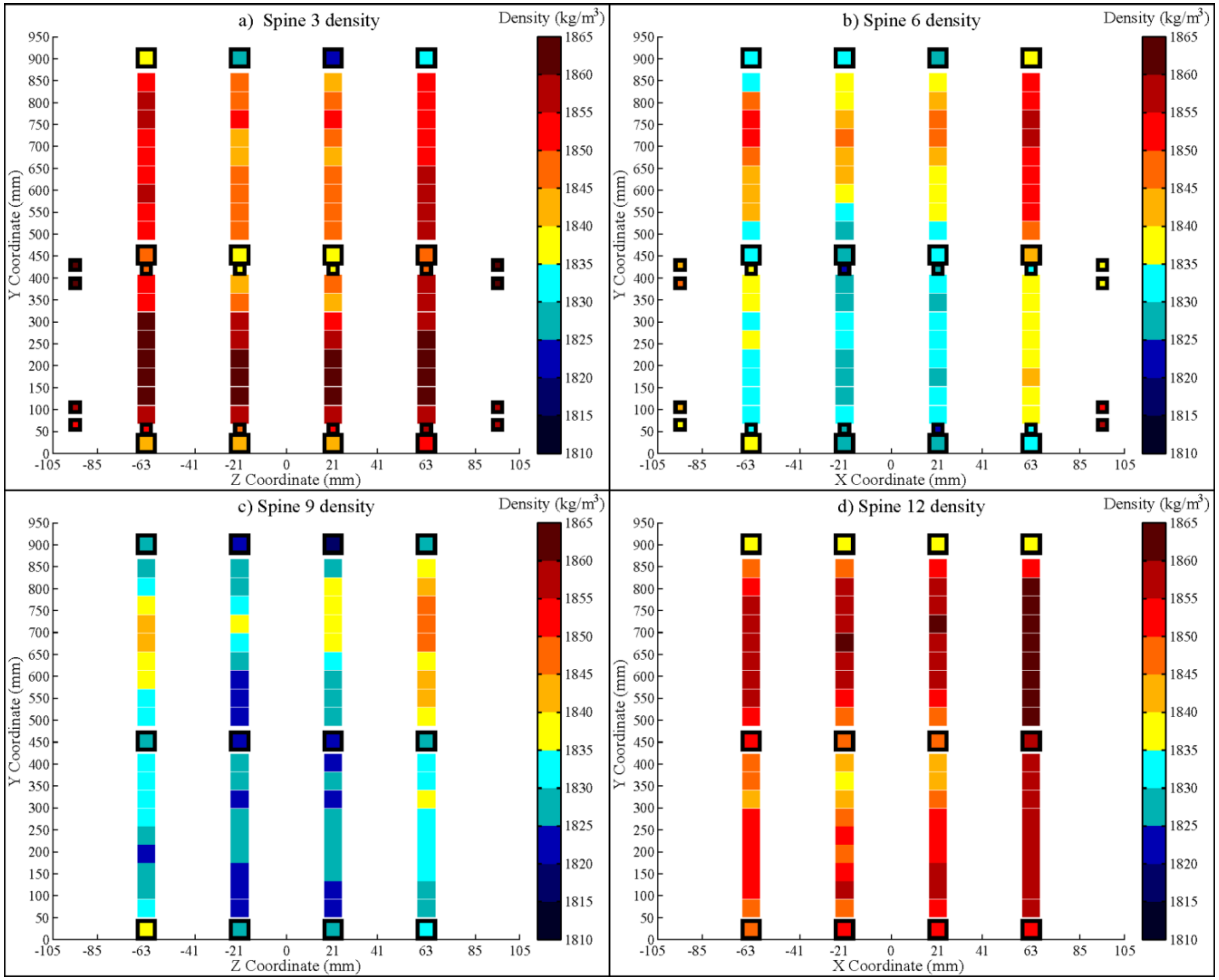
# Density


1810 kg/m3
1865 kg/m3
Plates density
Spines density
Frequency
Density (kg/m3)
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
1810-1815
1815-1820
1820-1825
1825-1830
1830-1835
1835-1840
1840-1845
1845-1850
1850-1855
1855-1860
1860-1865

# Density

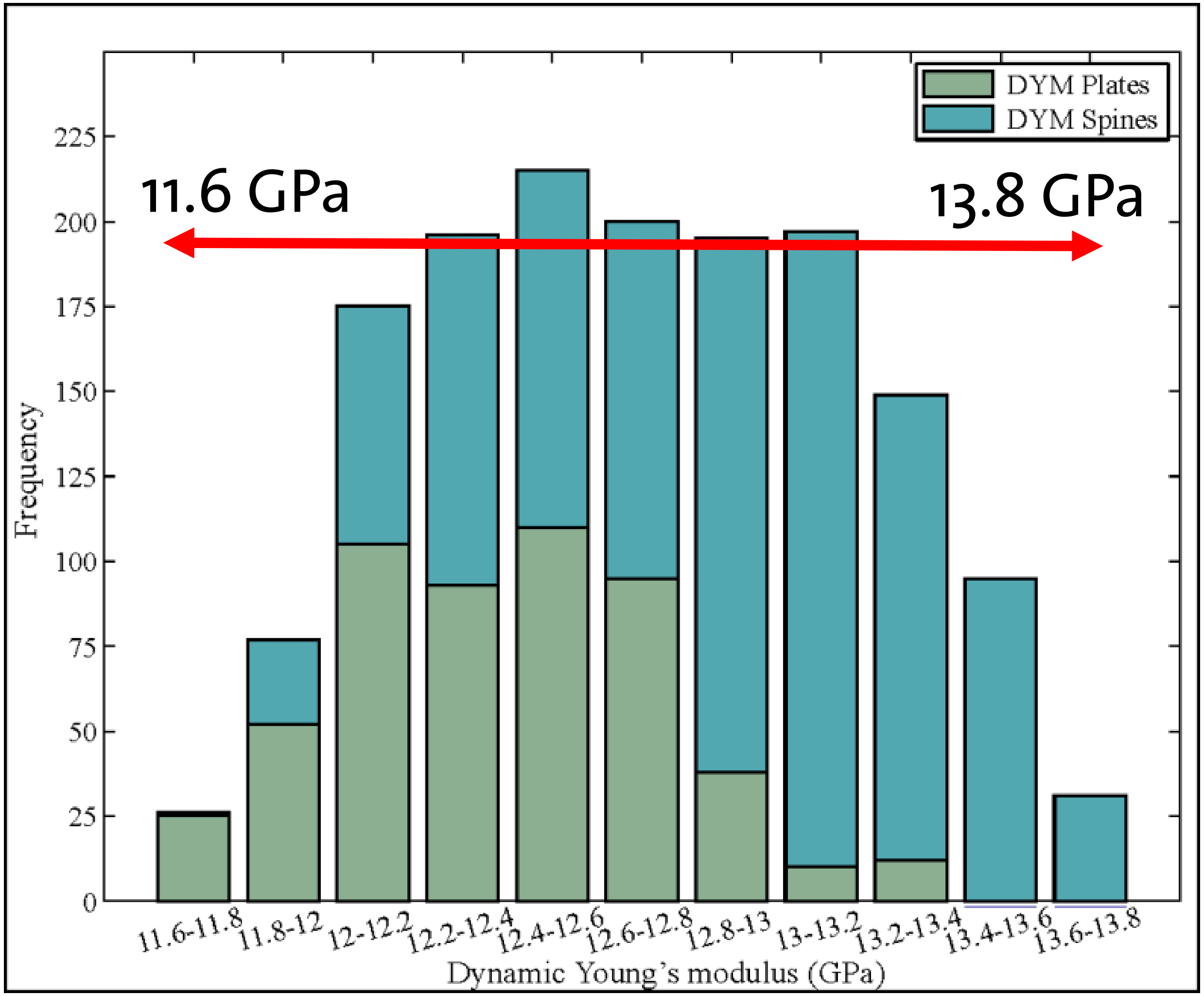

a) Density Plate A
b) Density Plate K
c) Density Plate U
Density (kg/m³)
Z Coordinate (mm)
X Coordinate (mm)
236 189 147 105 63 21 0 -21 -63 -105 -147 -189 -236
1865 1860 1855 1850 1845 1840 1835 1830 1825 1820 1815 1810
1860kg/m³
1835kg/m³
1810kg/m³

# Density

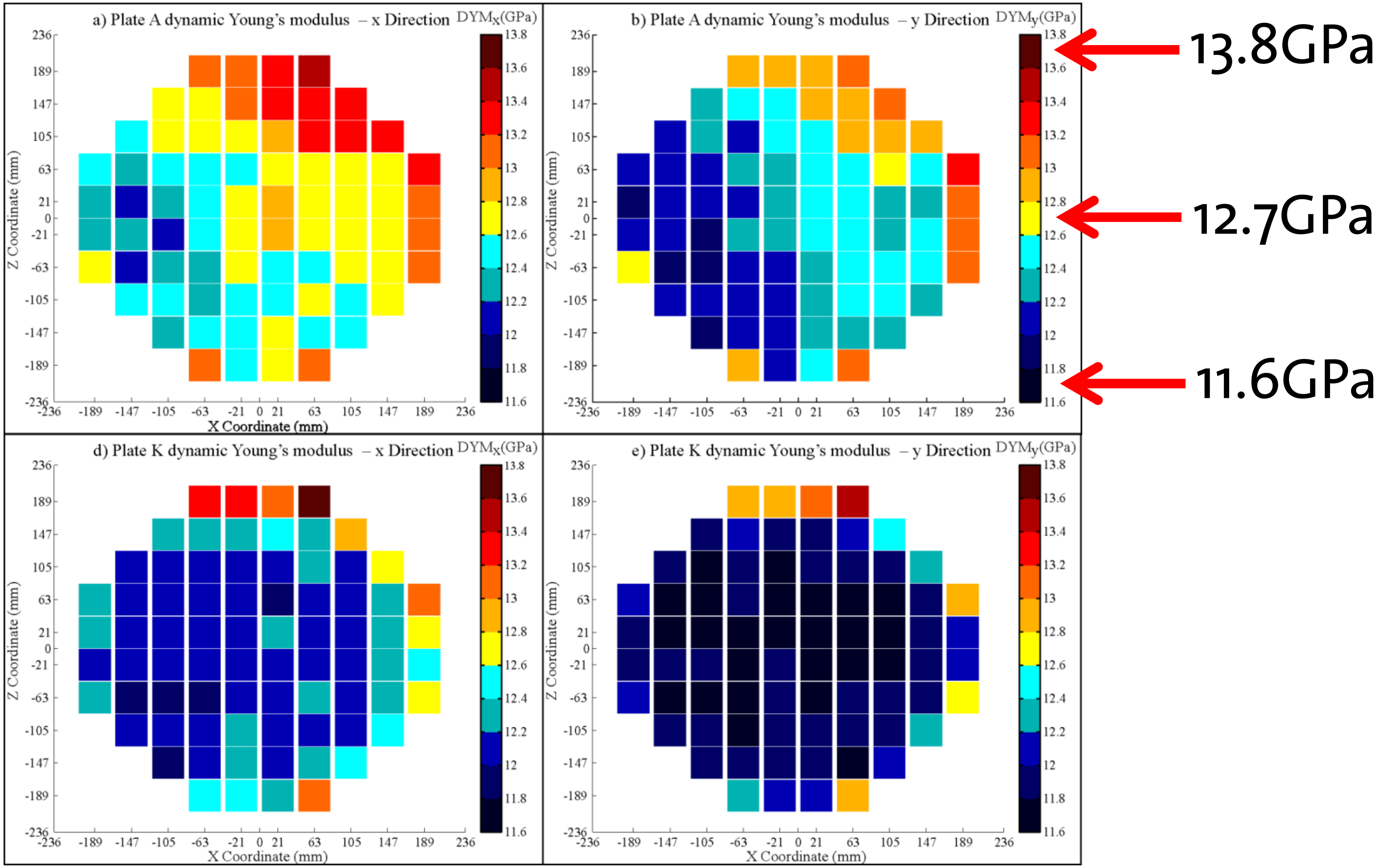


a) Spine 3 density
b) Spine 6 density
c) Spine 9 density
d) Spine 12 density
Density (kg/m$^3$)
1865
1860
1855
1850
1845
1840
1835
1830
1825
1820
1815
1810
Y Coordinate (mm)
Z Coordinate (mm)
X Coordinate (mm)
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
-105 -85 -63 -41 -21 0 21 41 63 85 105

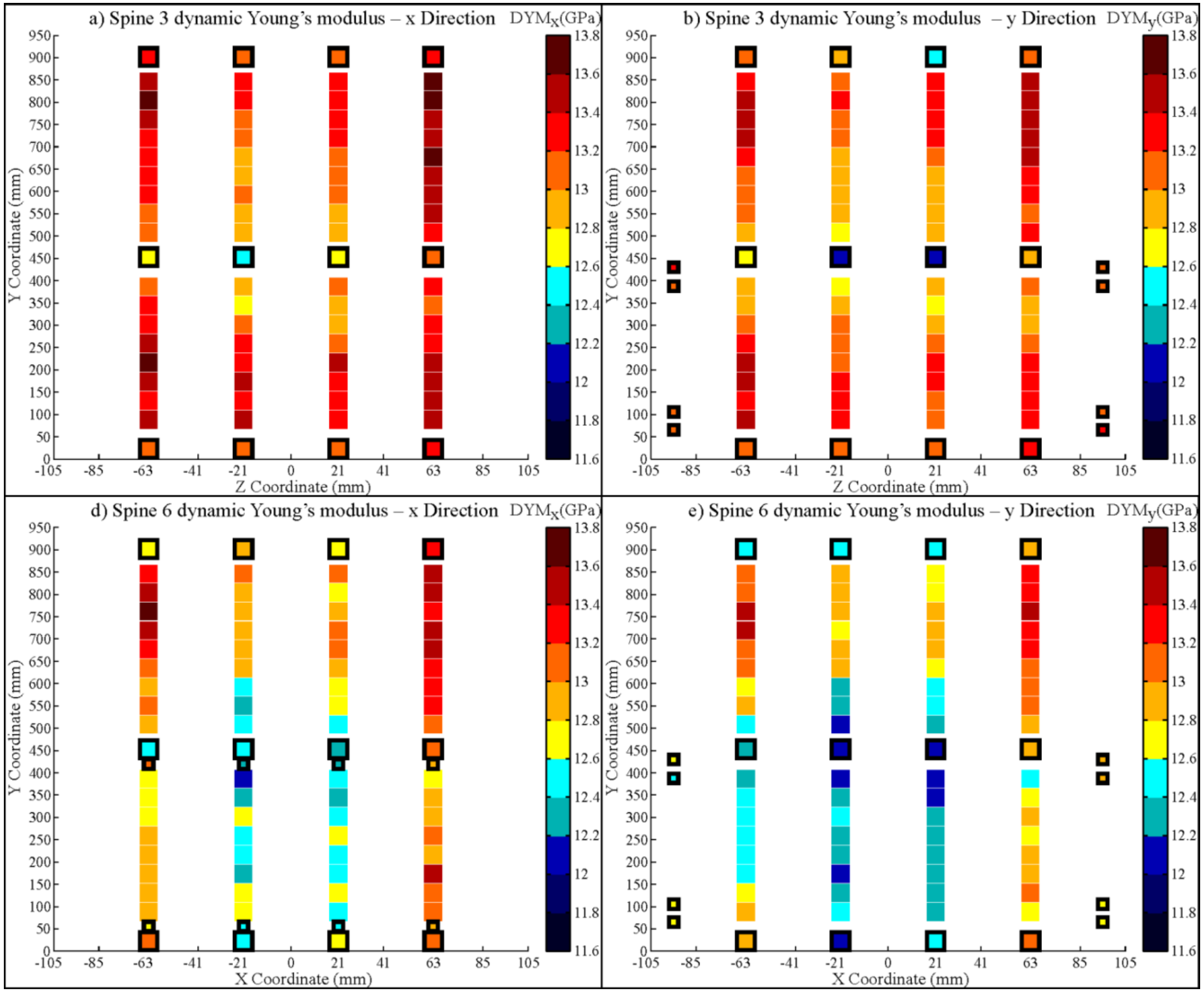
# Young's Modulus



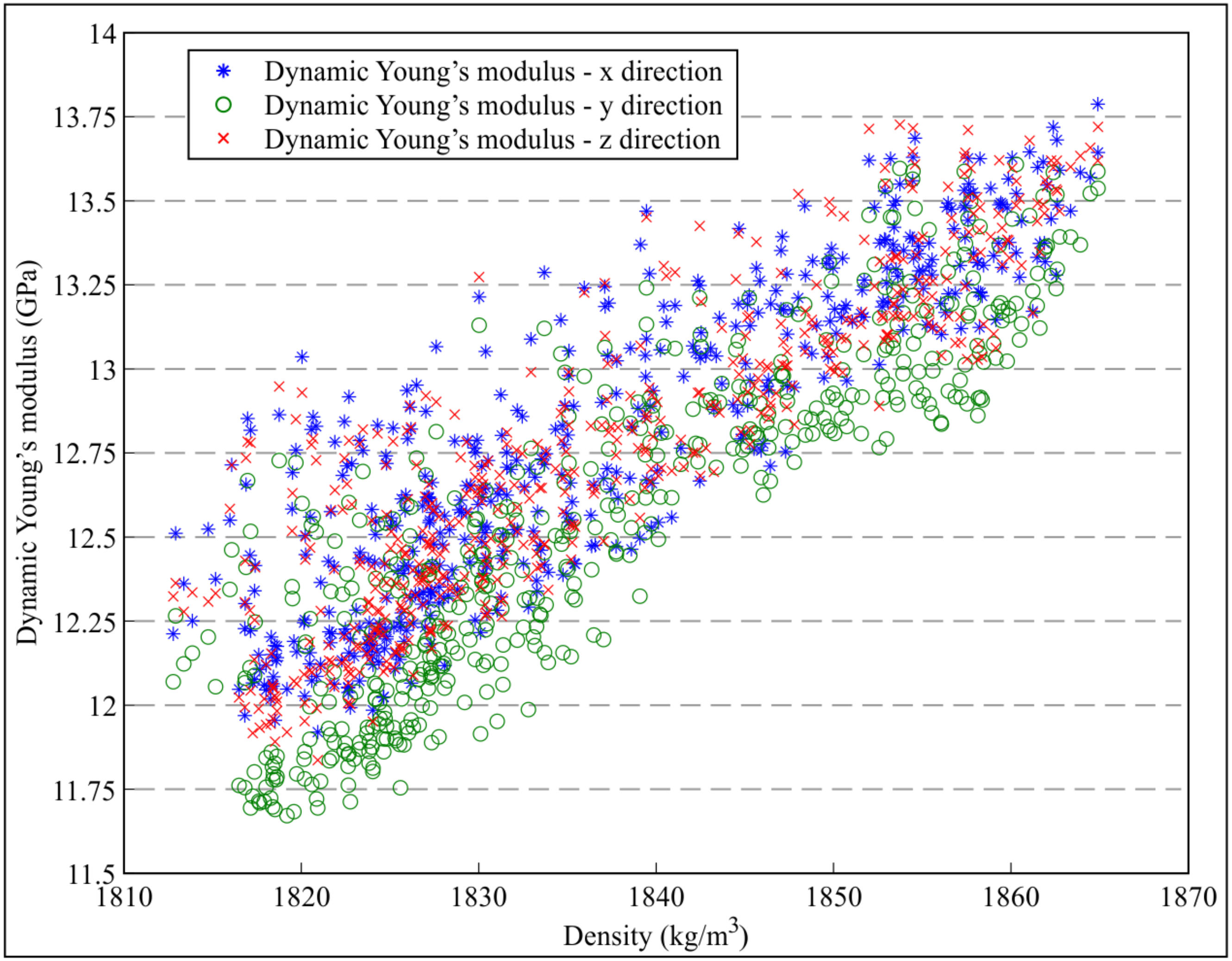
# Young's Modulus



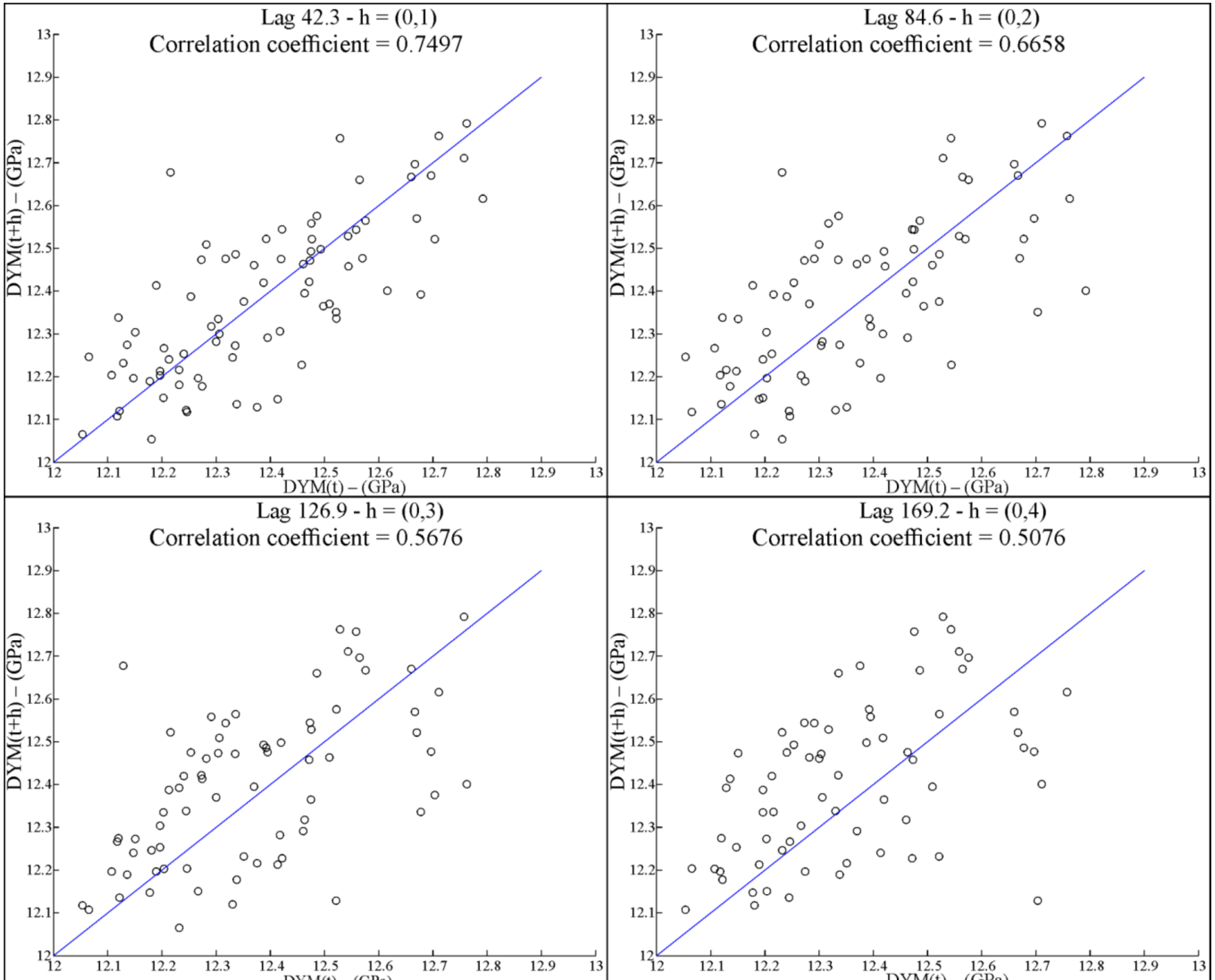
# Young's Modulus (Spines)



# Young's Modulus vs Density



# Spatial Correlation Length



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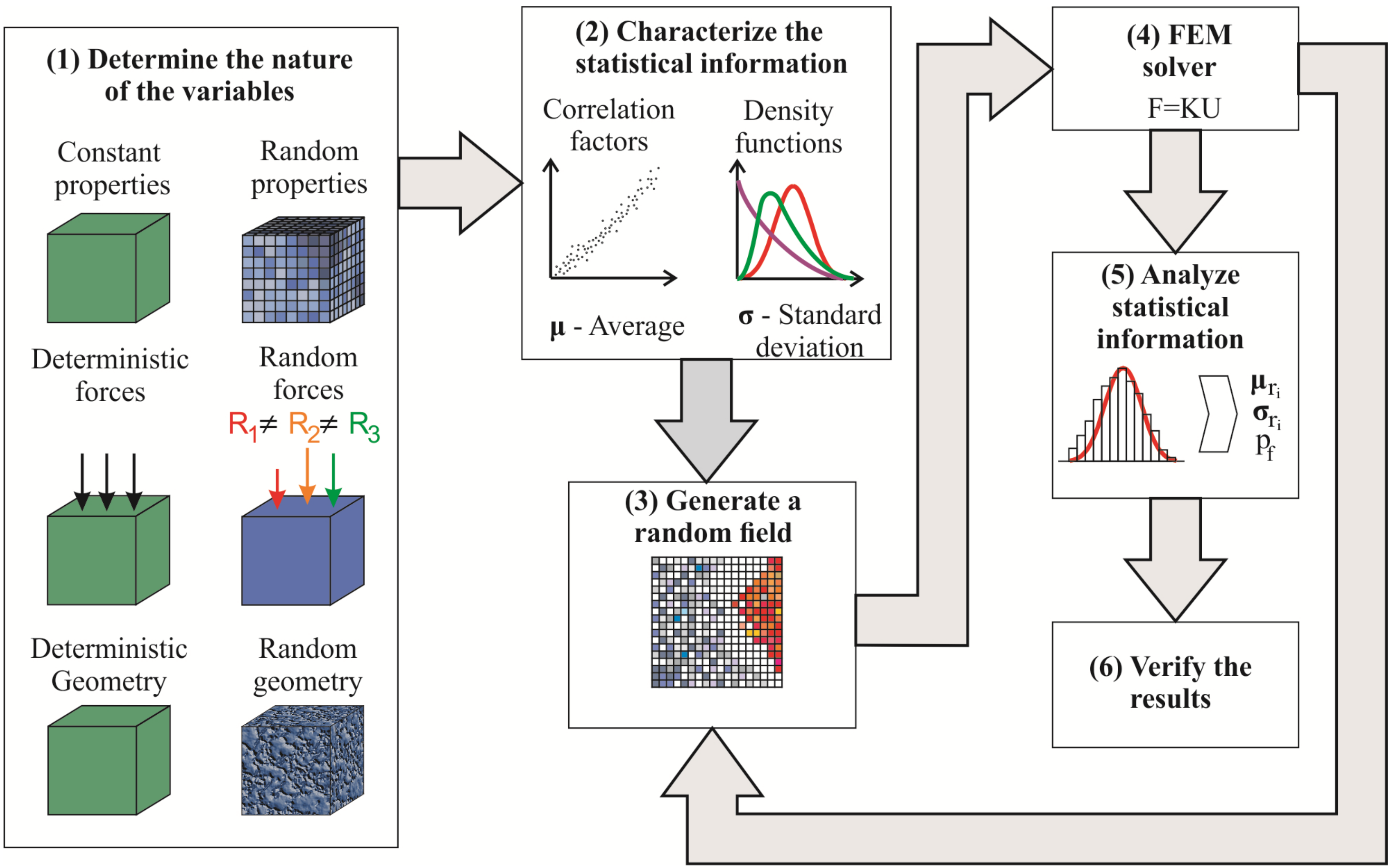
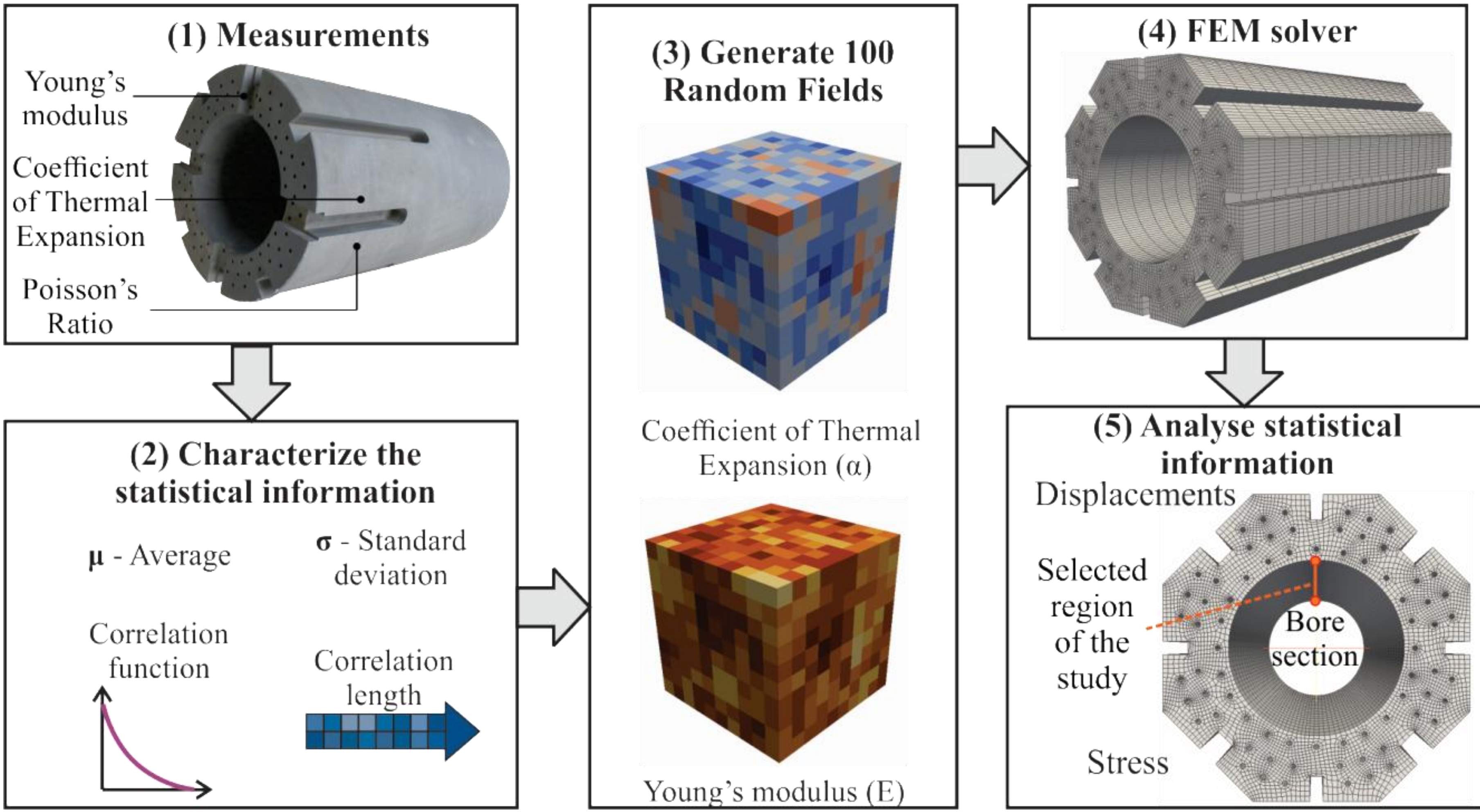
# Using the Data in a Finite Element Analysis

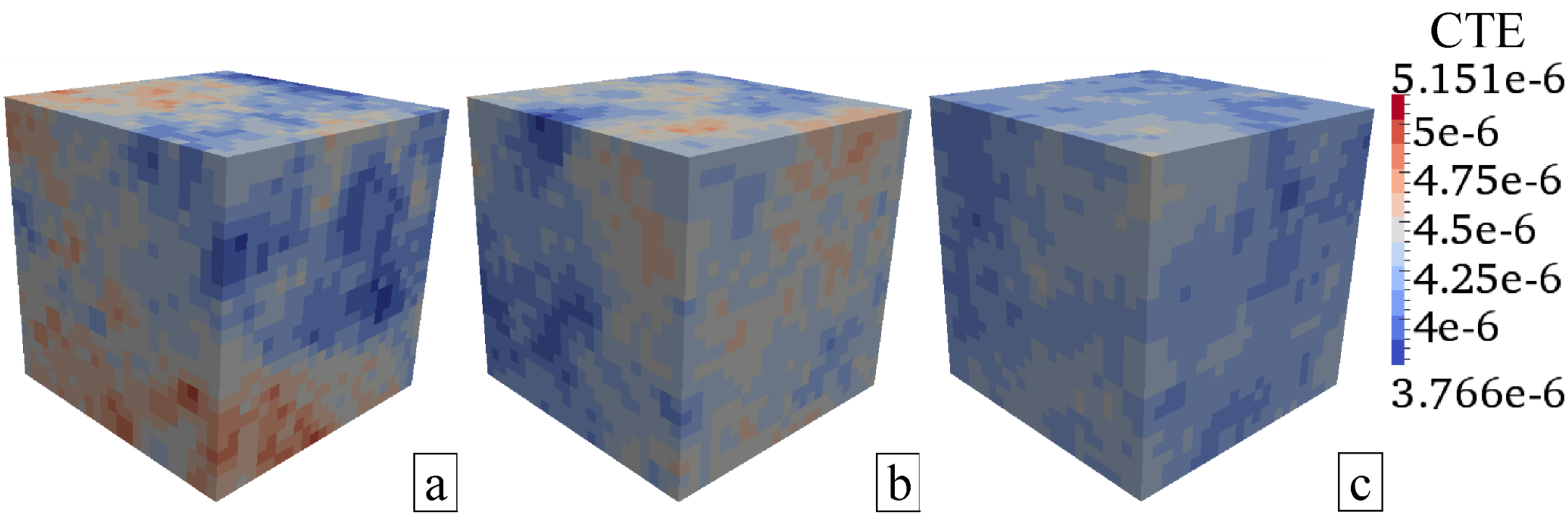
# General Workflow



# Workflow


(1) Measurements
Young's modulus
Coefficient of Thermal Expansion
Poisson's Ratio
(2) Characterize the statistical information
μ - Average
σ - Standard deviation
Correlation function
Correlation length
(3) Generate 100 Random Fields
Coefficient of Thermal Expansion (α)
Young's modulus (E)
(4) FEM solver
(5) Analyse statistical information
Displacements
Selected region of the study
Bore section
Stress

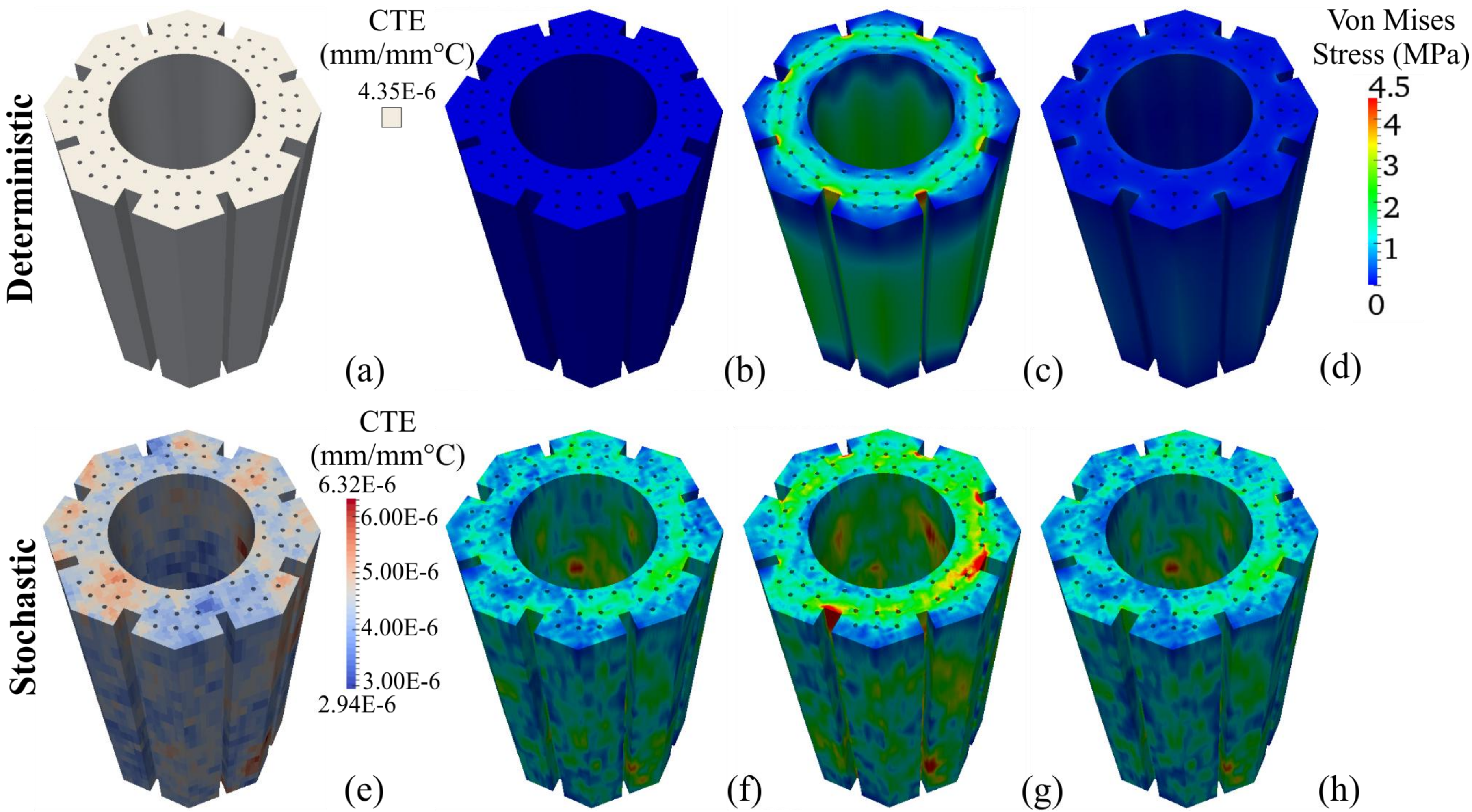
# Effect of Correlation Length




Correlation length of 12 mm

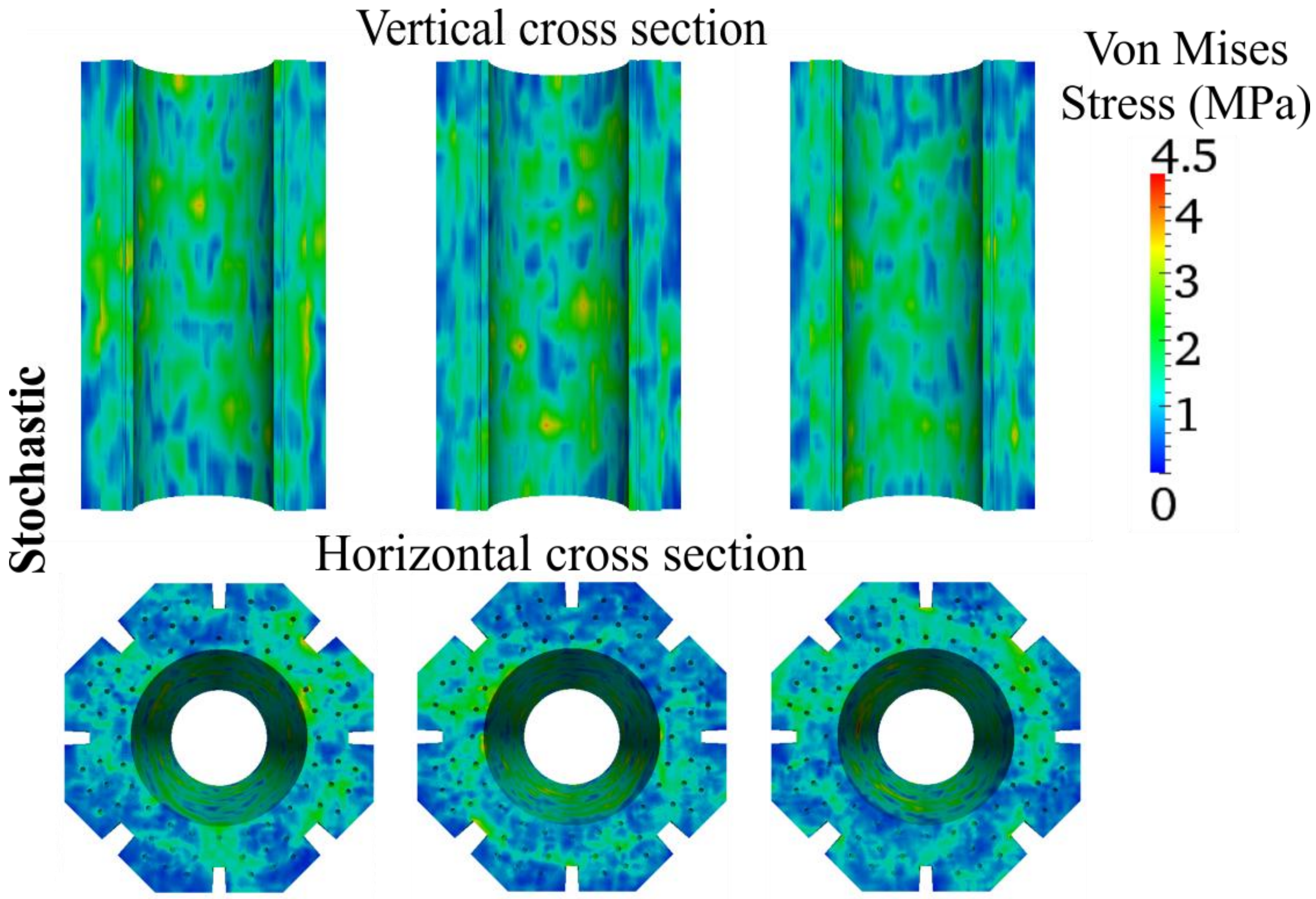
Correlation length of 24 mm

Correlation length of 48 mm



Deterministic
CTE
(mm/mm°C)
4.35E-6
(a)
(b)
(c)
(d)
Von Mises
Stress (MPa)
4.5
4
3
2
1
0
Stochastic
CTE
(mm/mm°C)
6.32E-6
6.00E-6
5.00E-6
4.00E-6
3.00E-6
2.94E-6
(e)
(f)
(g)
(h)

# Three Analyses



# Summary

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- Thermomechanical analysis
  - It is usually assumed unconstrained expansion due to a temperature change does not result in the generation of stresses
  - In this example, tiny variations in the material properties of a material designed to be uniform and isotropic (small coefficient of variation) generate stresses of up to ~5MPa
  - This occurs even before the reactor is in service. So the initial conditions used in a deterministic analysis are "wrong".
  - These may influence where and when failure will occur
  - Further finite element analysis is required to predict where and when failure will probably occur – towards probabilistic design

# Summary

- Dealing with uncertainty
  - Can incorporate real-world uncertainties into engineering analysis, allowing their effect to be known, managed and controlled if possible.
  - Provides the basis for improving the safety and quality of engineered systems through risk-informed design practice.
  - Can be used to develop rational strategies for pricing, warranties, inspections, availability, maintenance, etc.

# Further Reading

- On the pen drive, there are four associated journal papers:
  1. A practical review of the stochastic finite element method.
  2. Proof-of-concept analysis for the thermo-mechanical case study.
  3. Experimental work to determine the spatial variability of graphite.
  4. Conversion of experimental data into random variables.