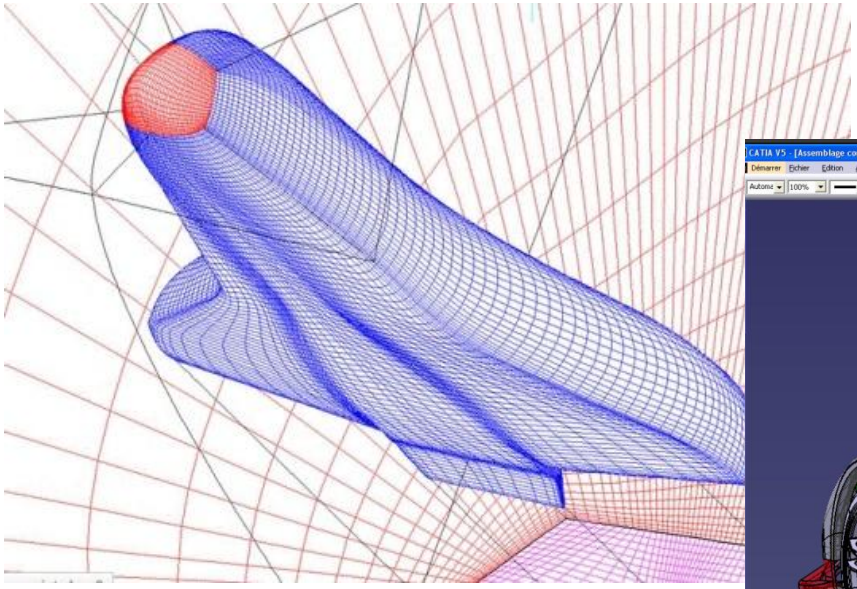
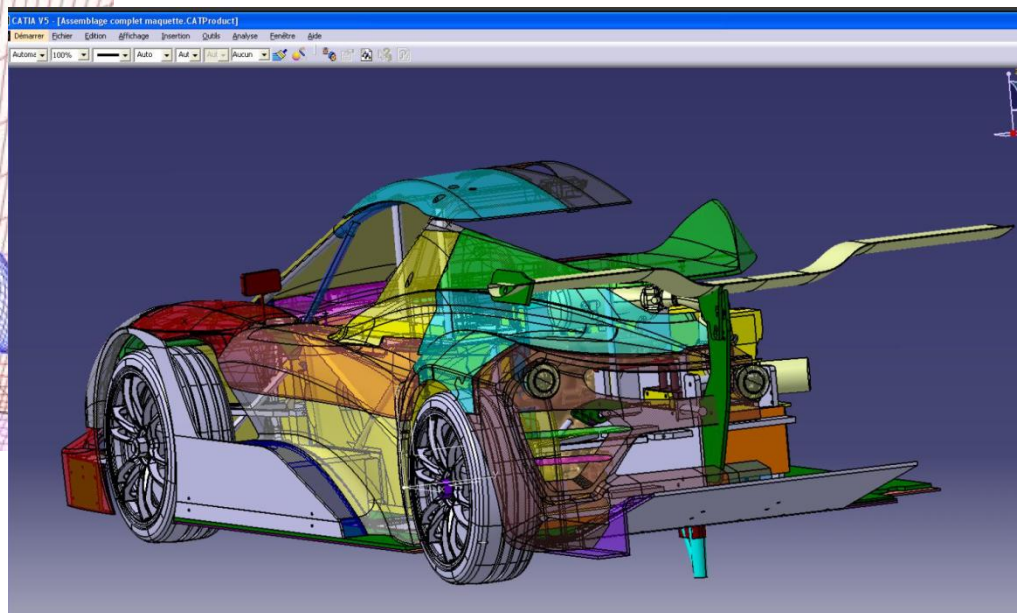
September 5 2018

Mathematical Morphology based EF mesh

Franck N'Guyen

Computer-aided design (CAD)





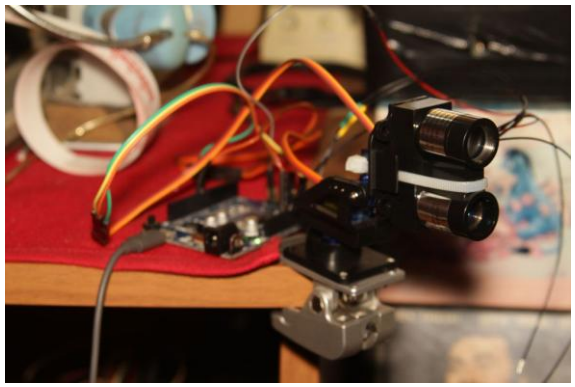
CATIA
SolidWorks
Etc ….

Image acquisition

Tomography X

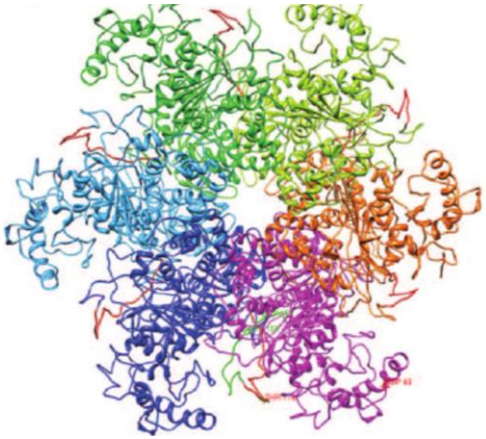
Confocal microscopy

MRI

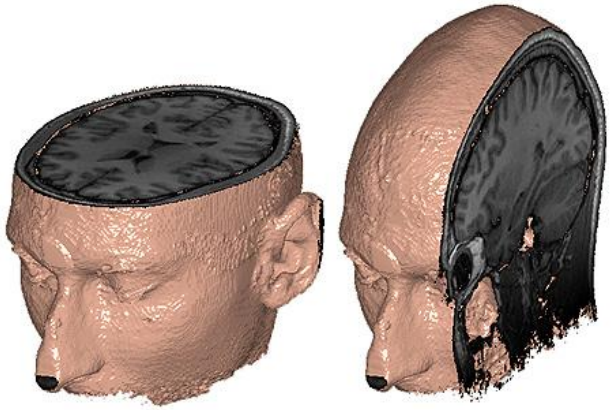


LIDAR laser

The microstructure

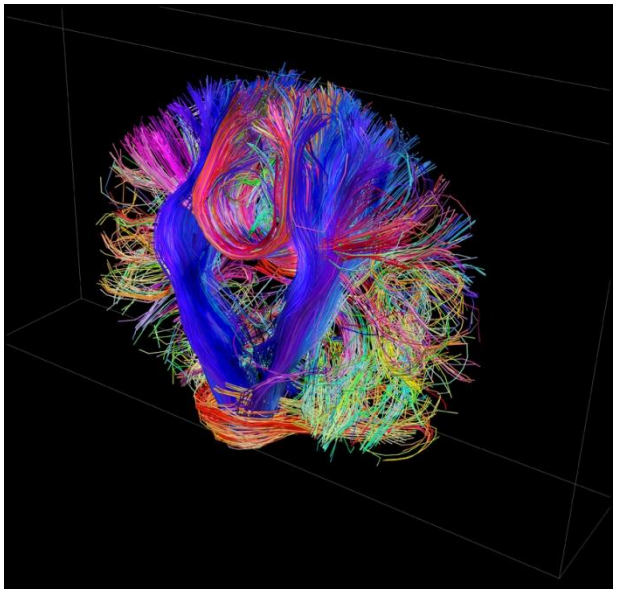


Protein



MRI brain

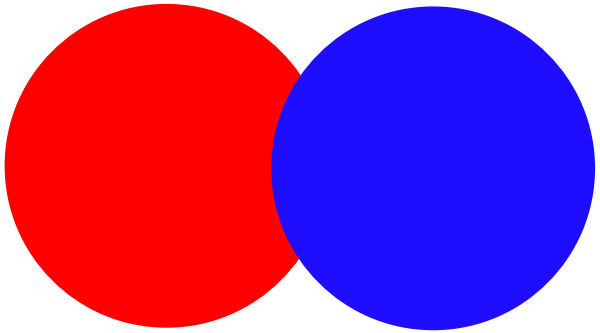
Tomographic
3D image



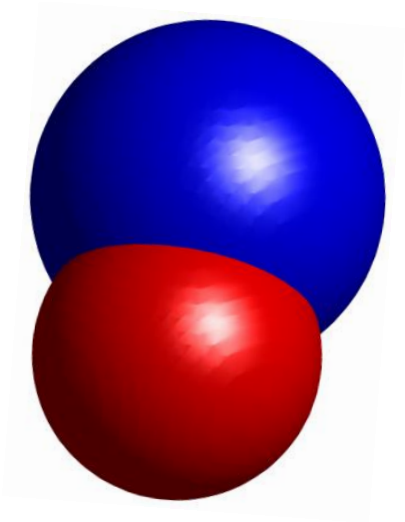
neuronal connection of a brain

Study of the thermo-mechanical behavior ?

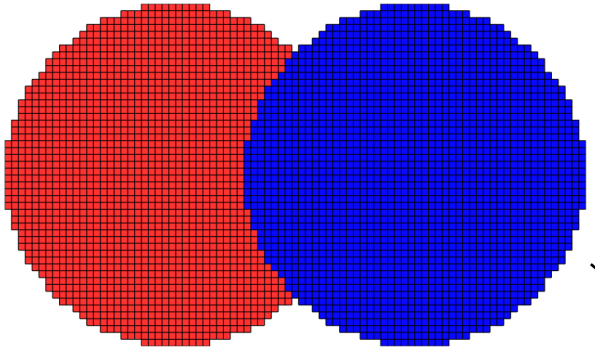
A discretized world



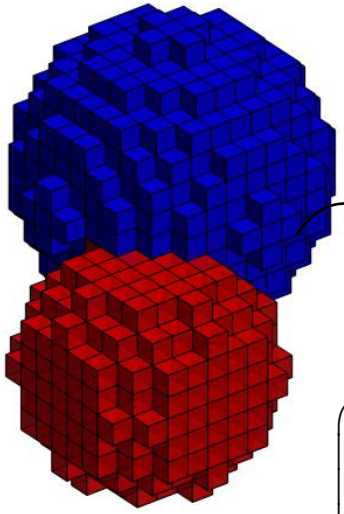
$$x^2 + y^2 = r^2$$



$$x^2 + y^2 + z^2 = r^2$$



$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

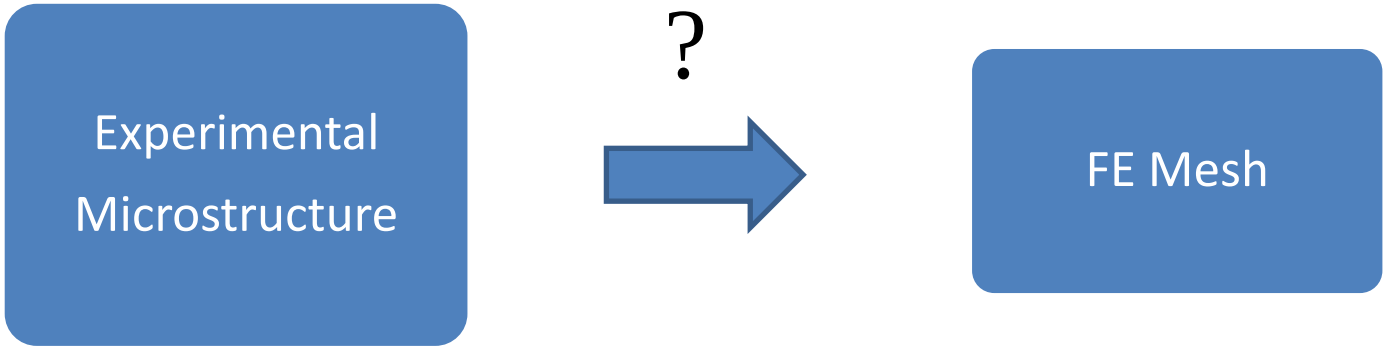


$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad z+1$$

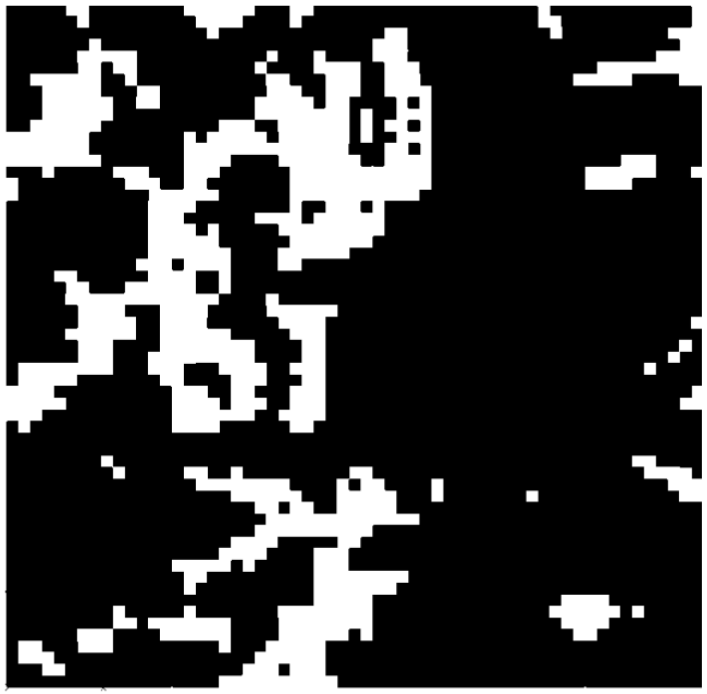
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad z$$

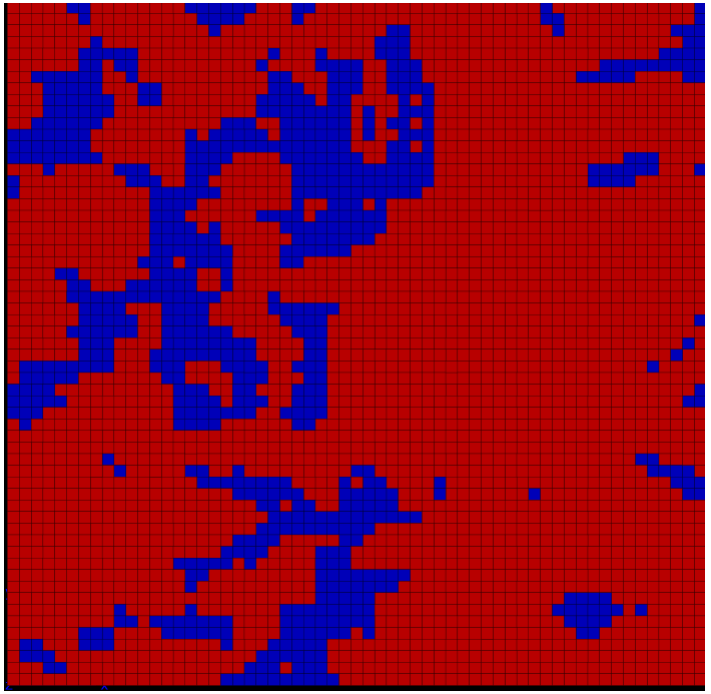
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad z-1$$

What is the problem ?



Natural mesh

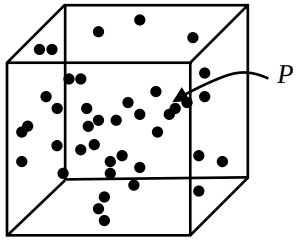




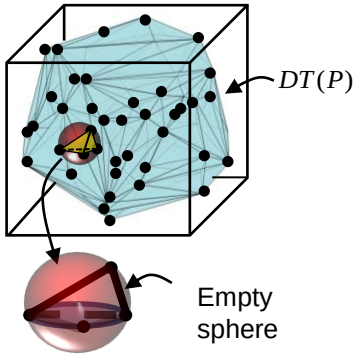
1 Pixel/Voxel = 1 FE mesh

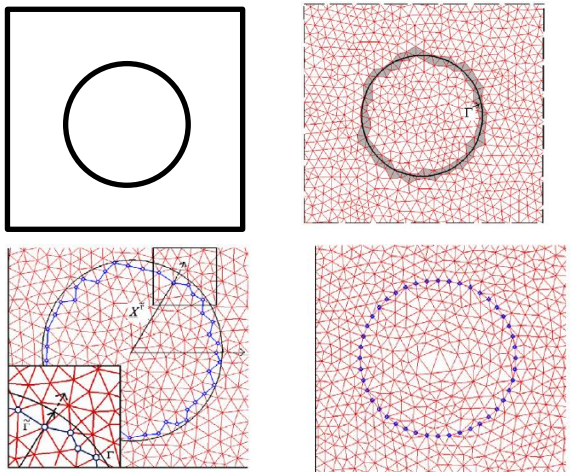
Quick overview

Geometrical approach

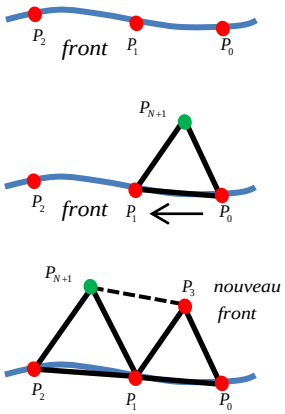


Delaunay triangulation

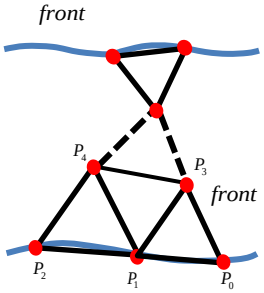


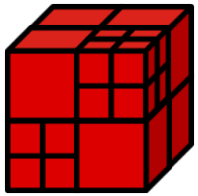


Nodes replacement



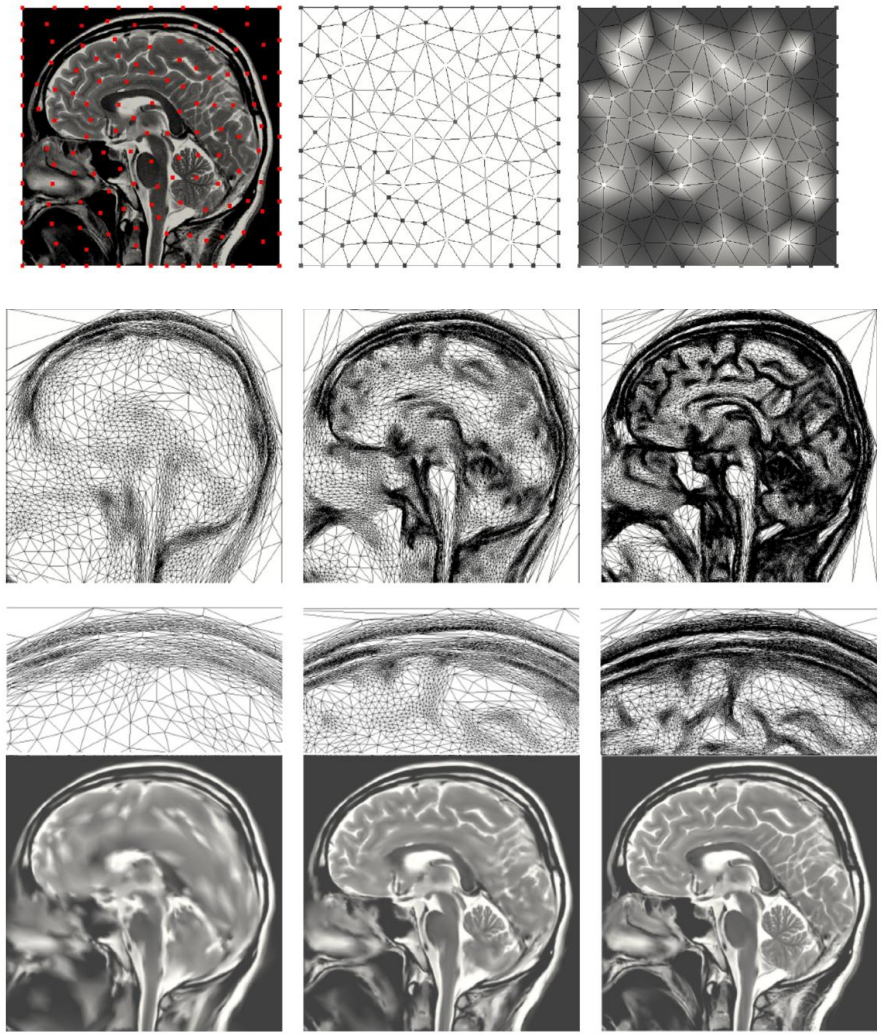
Frontal mesh





Octree based mesh

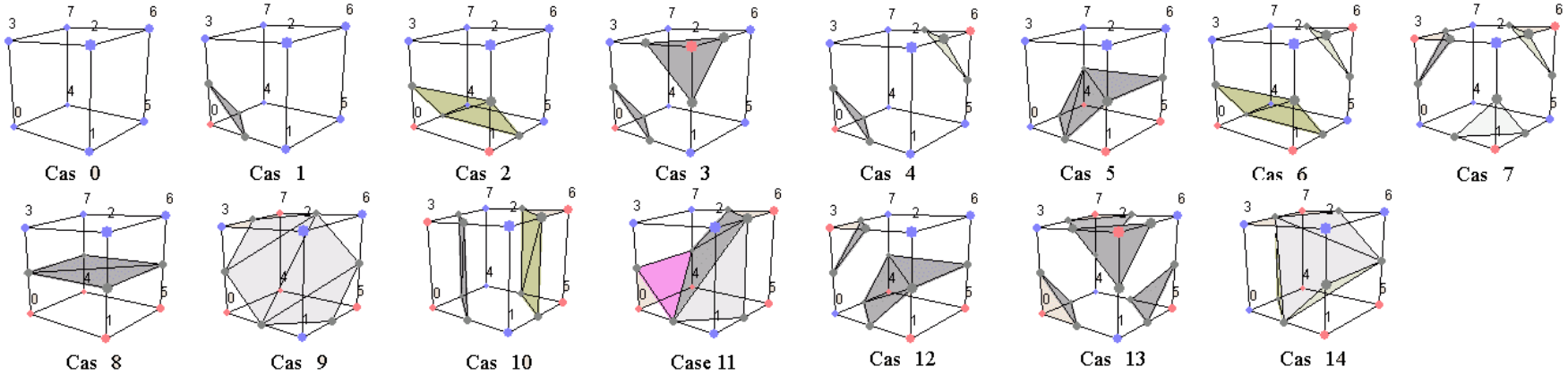
Morphing image/Mesh

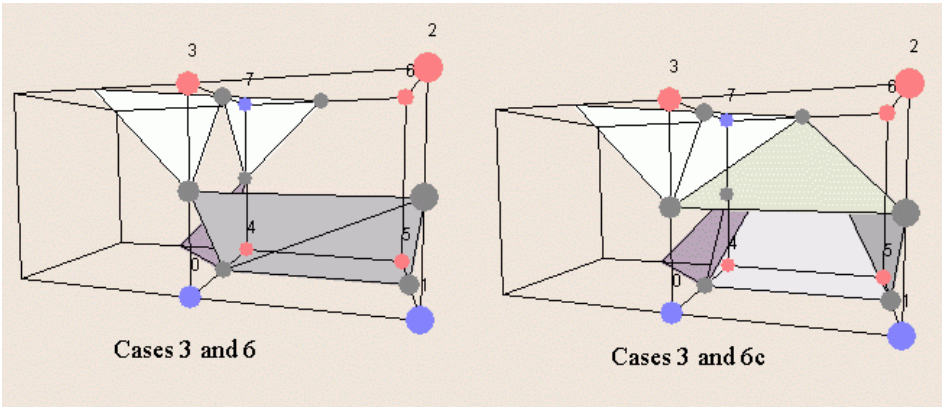


OOF 2D / OOF3D
S.Langer

“Direct multiphase mesh generation from 3D images using anisotropic mesh adaptation and a redistancing equation” Jia-Xin Zhaoa and al

Marching cube

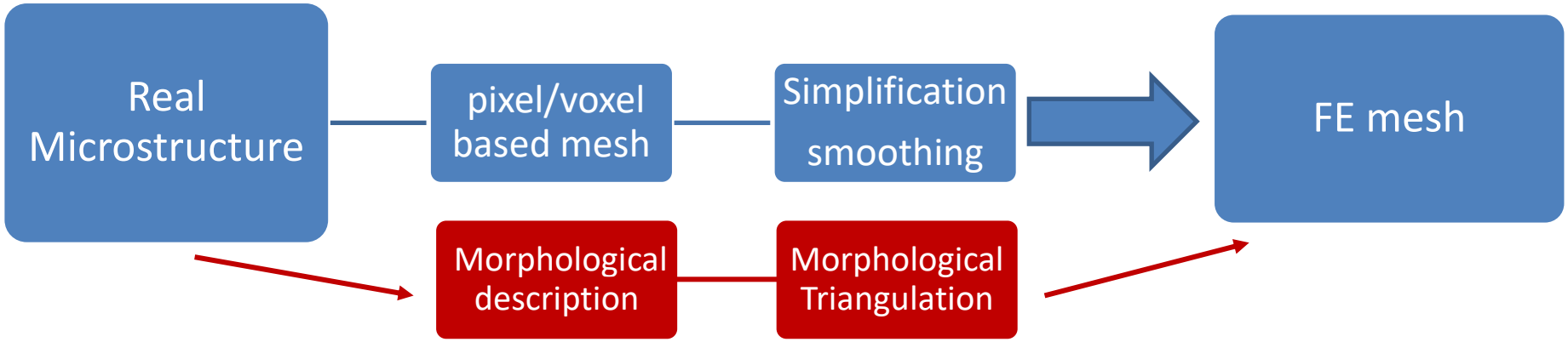




Presence of holes

Chosen strategy

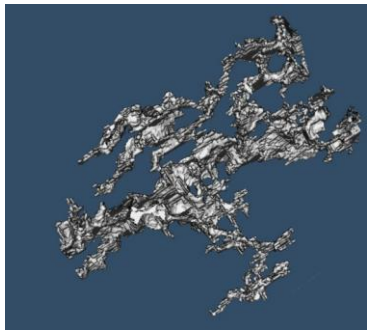
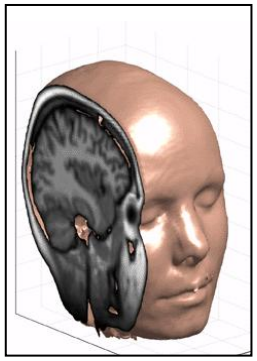
Alternative to a systematic partition of a set of pixels / voxels



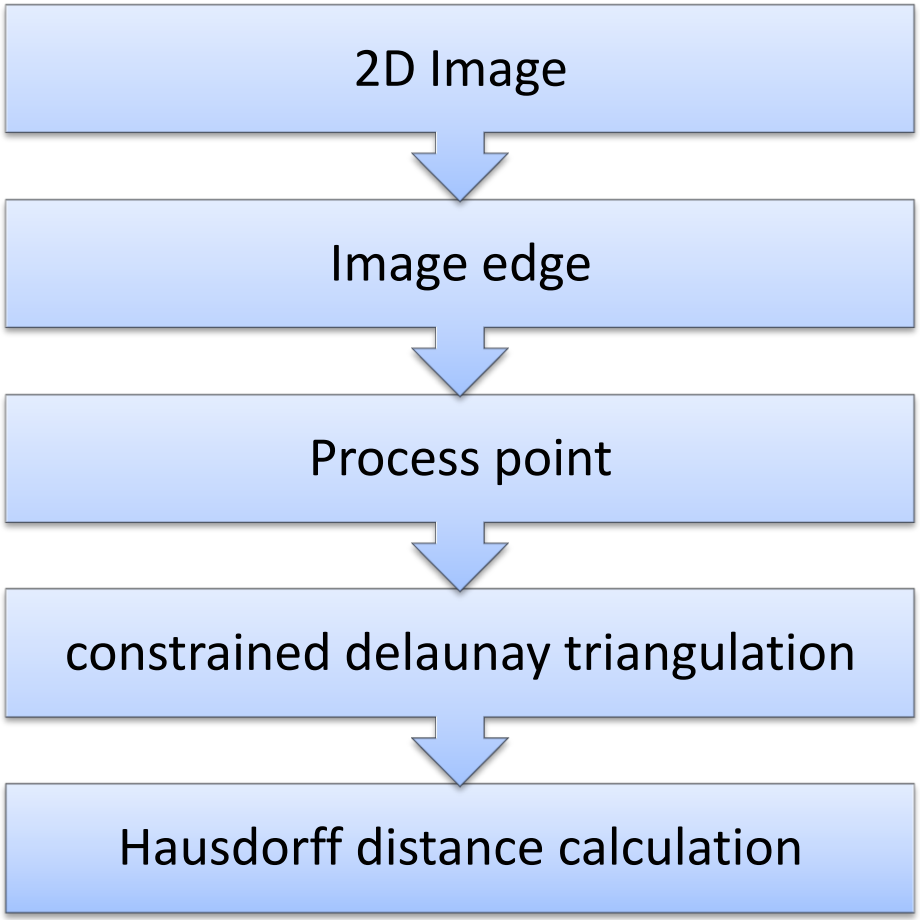
Microstructure morphology consideration whith Mathematical Morphologie concepts

Significantly reduces the number of elements and bypass the decimation step while respecting the morphology of the microstructure

Images to be processed

	X	f
$2D$		
$3D$		

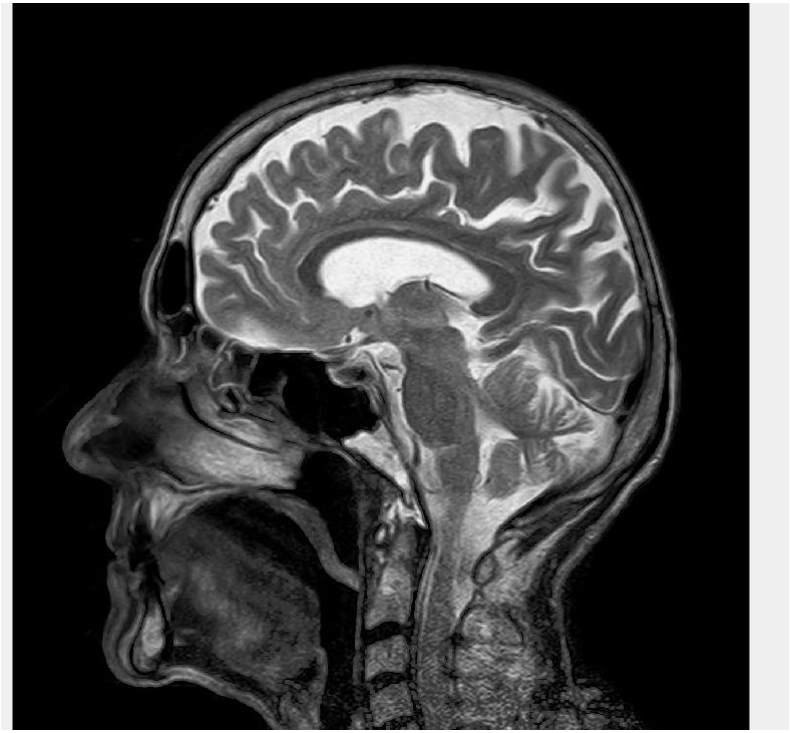
Organigram


2D Image
Image edge
Process point
constrained delaunay triangulation
Hausdorff distance calculation

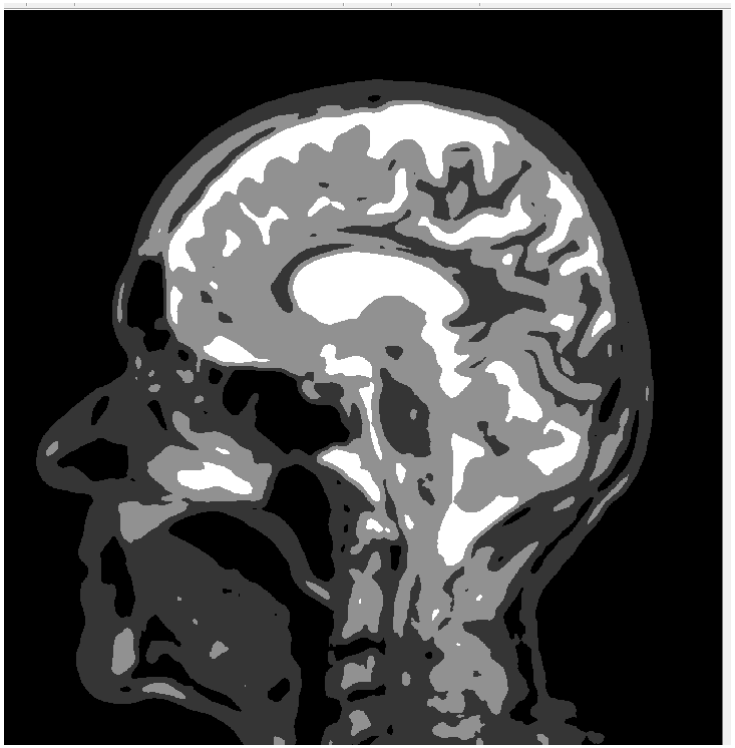
Binary 2D images

Enriched segmentation

Method : Particle swarm optimization (PSO)



original image

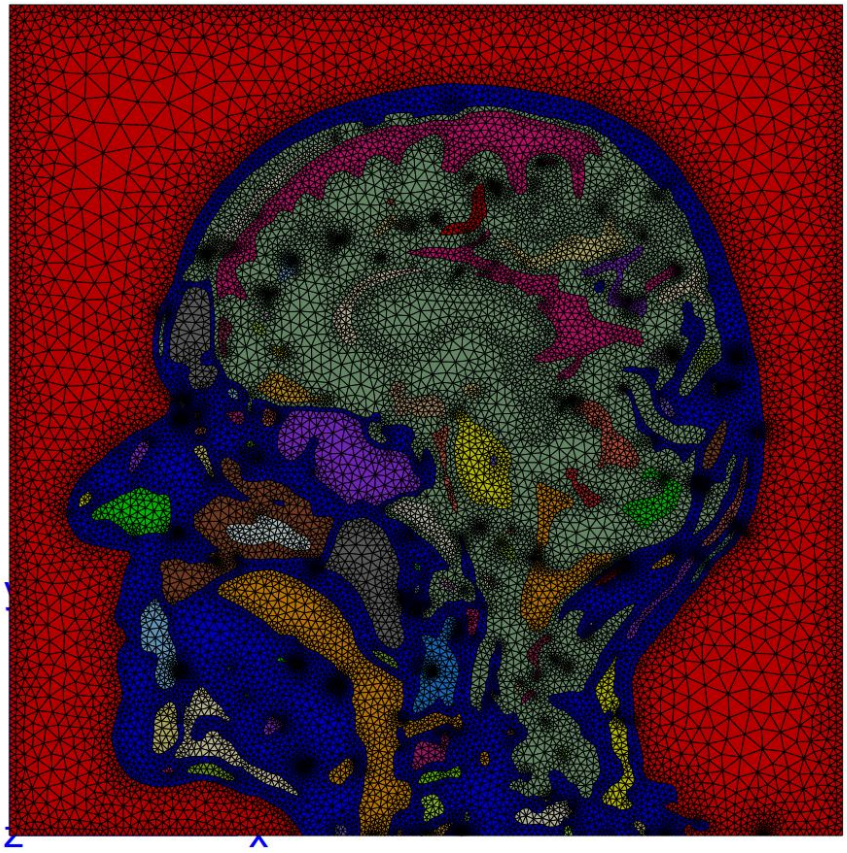


Segmentation in clusters of gray levels

2D mesh

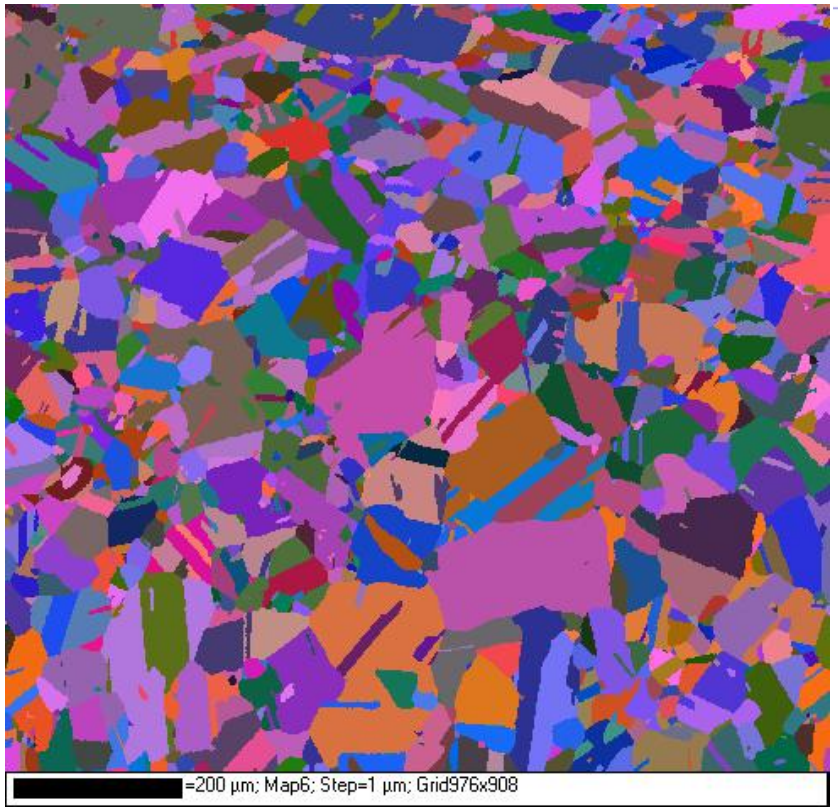
$$\nabla_B(f) = \partial(f) = \delta_B(f) - \mathcal{E}_B(f)$$

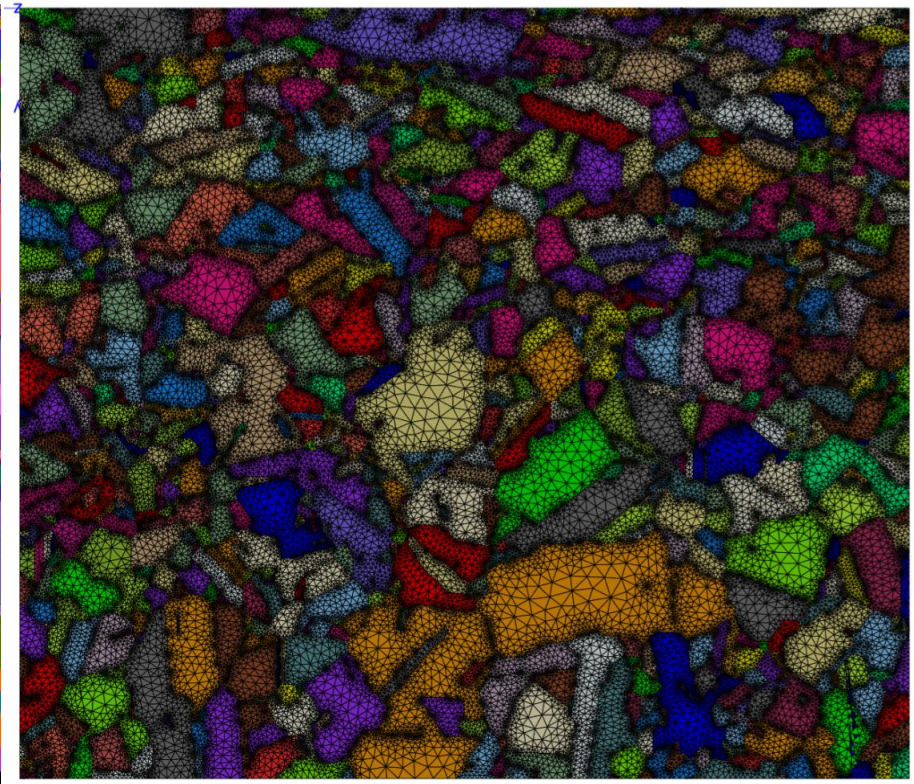
Morphological gradient functional case

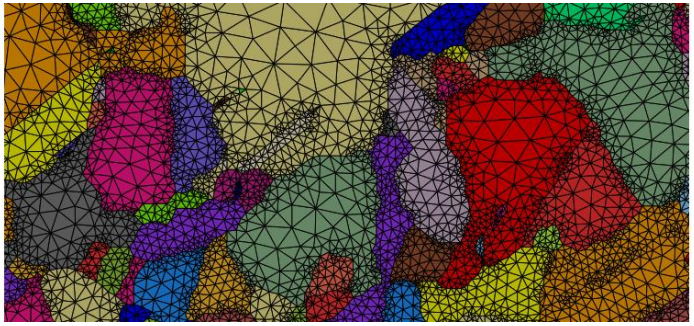


Delaunay-refinement and Frontal-Delaunay triangulation techniques. D. Engwirda

Other result

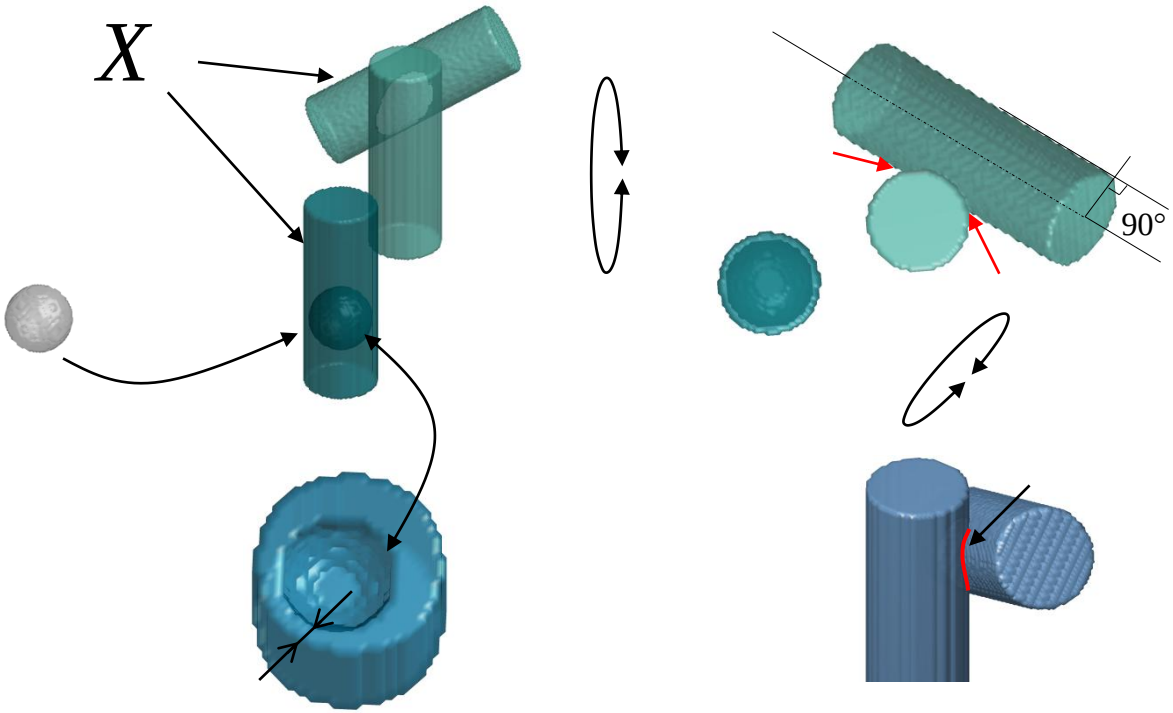




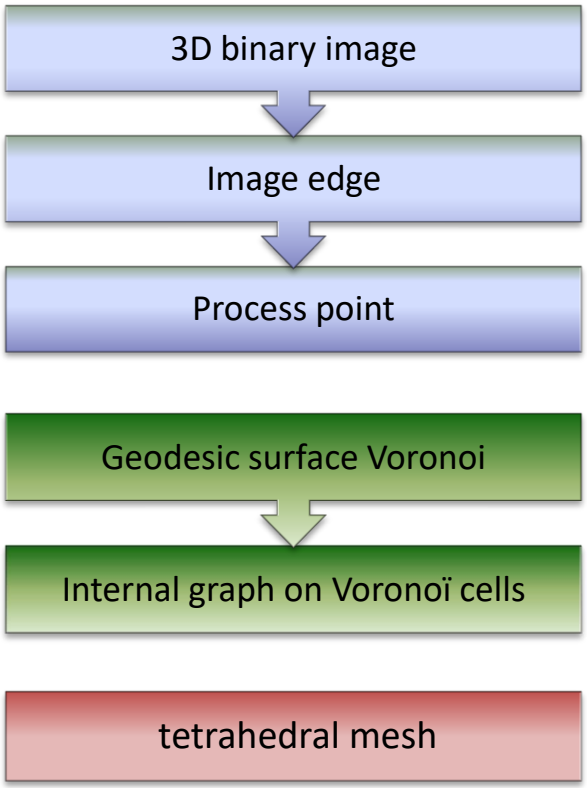


No morphological compromise

Binary 3D images



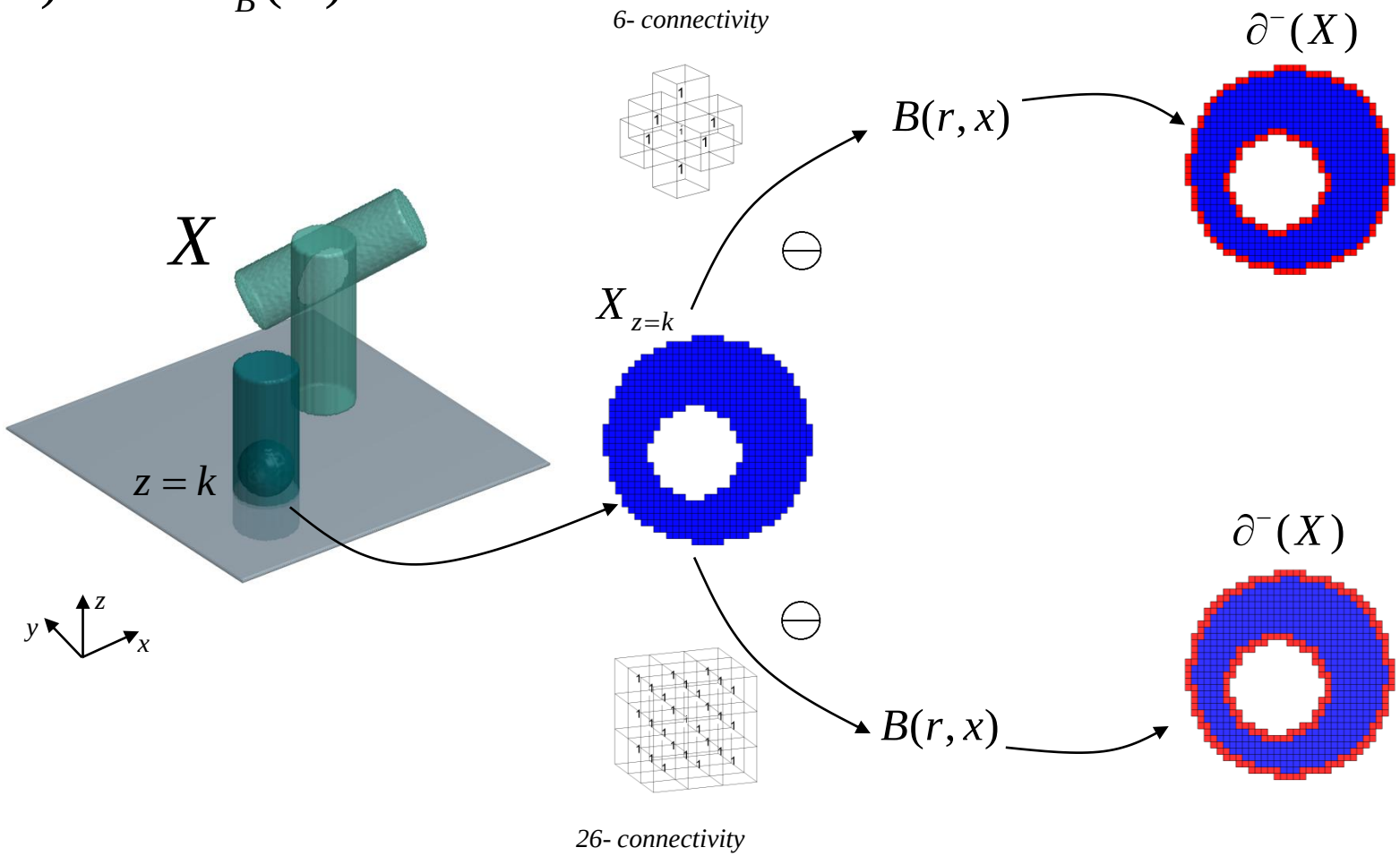
Organigram


3D binary image
Image edge
Process point
Geodesic surface Voronoi
Internal graph on Voronoï cells
tetrahedral mesh

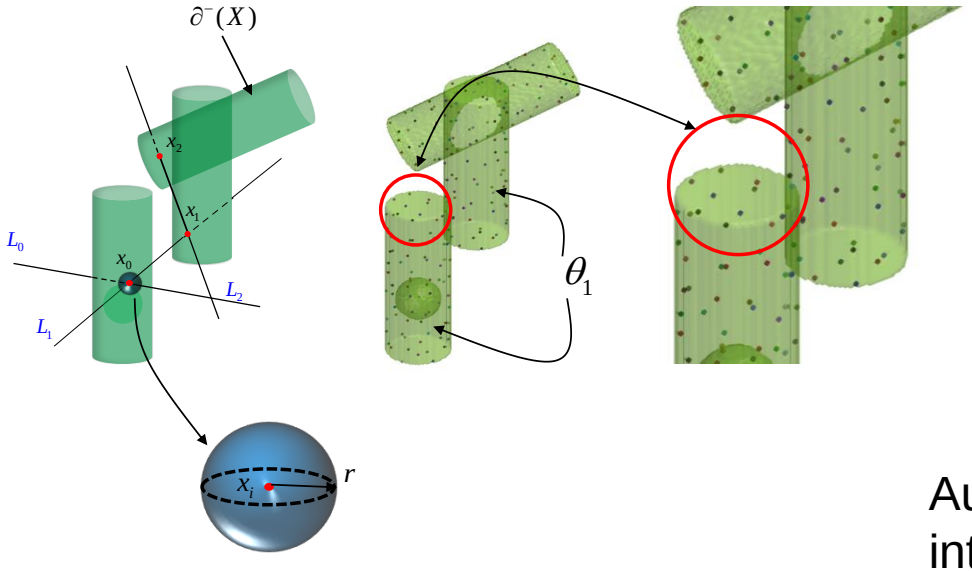
Surface of a binary 3D image

Internal Gradient : ensemblist case

$$\partial^{-}(X) = X \setminus \varepsilon_B(X)$$

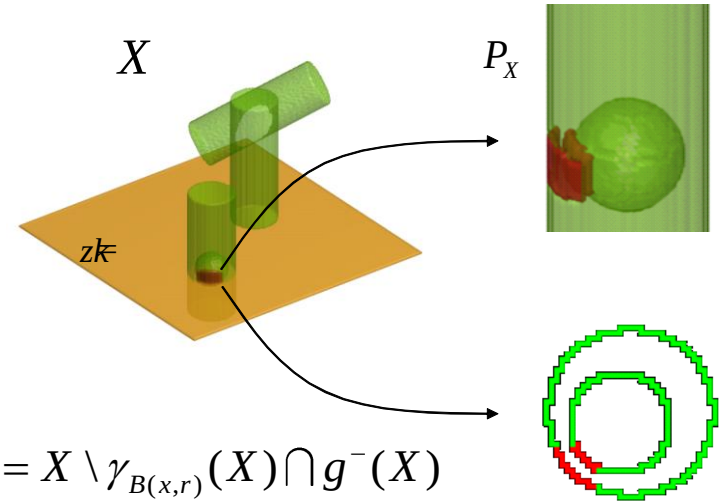


Point process

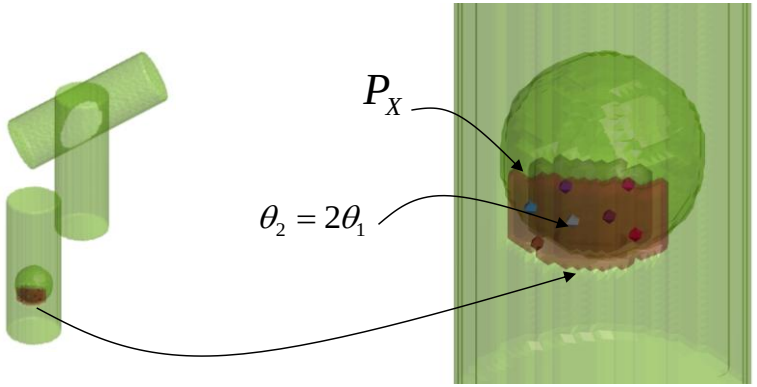


3D « *hit and run* » implantation process point (C.Lantuejoul)

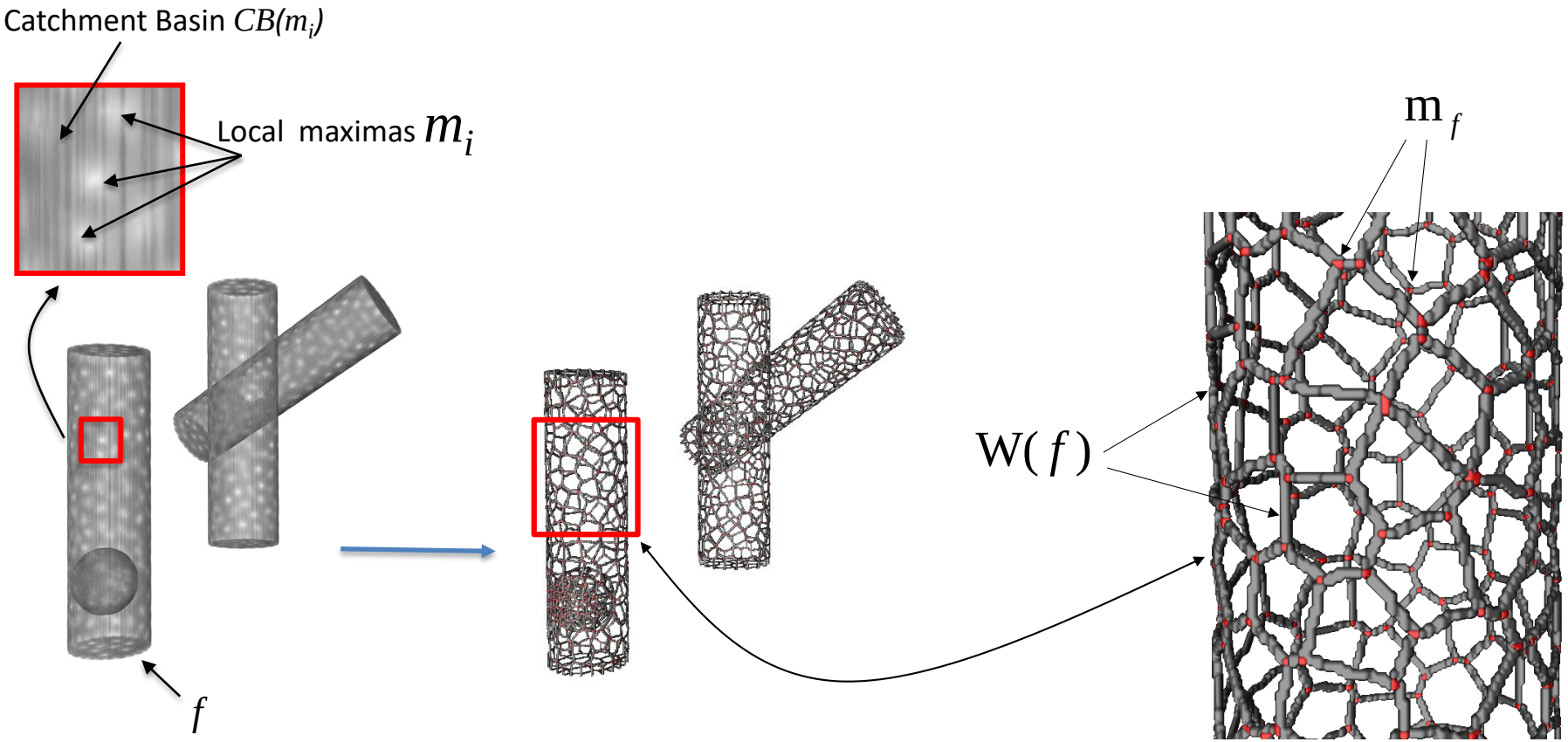
Automatic morphological intensity point process control



$$P_X = X \setminus \gamma_{B(x,r)}(X) \cap g^-(X)$$



Geodesic watershed

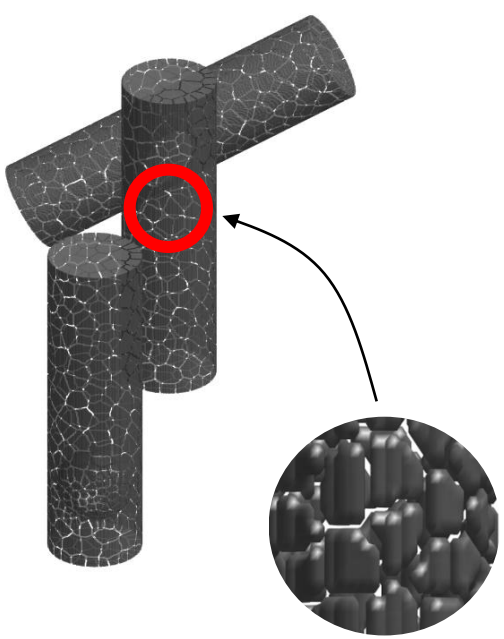


Geodesic distance function

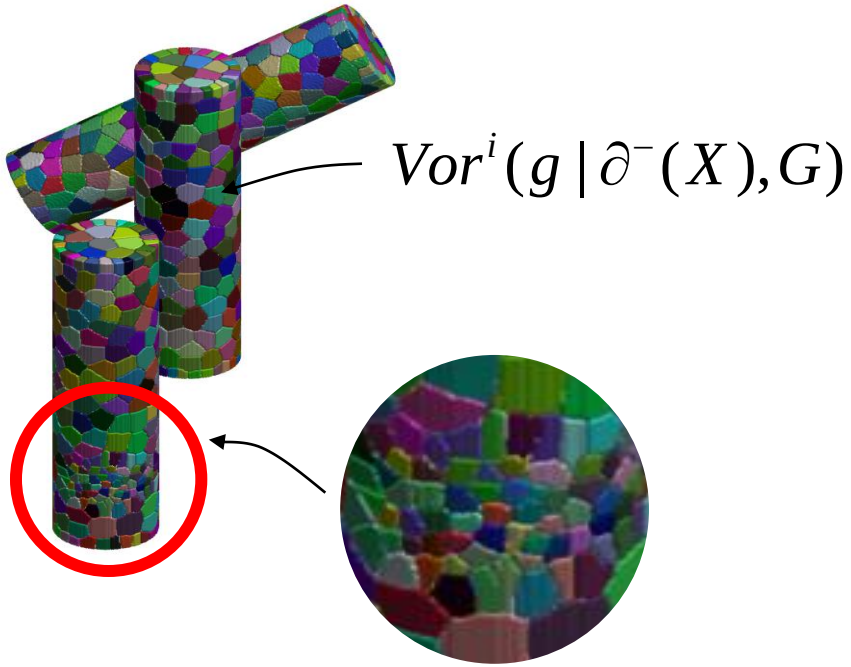
$$W(f) = sup(f) \cap \left[\cup \left(CB_i(m_i) \right) \right]^c$$

Geodesic Voronoï

$$M_X = \partial^-(X) \setminus \mathrm{W}(f)$$

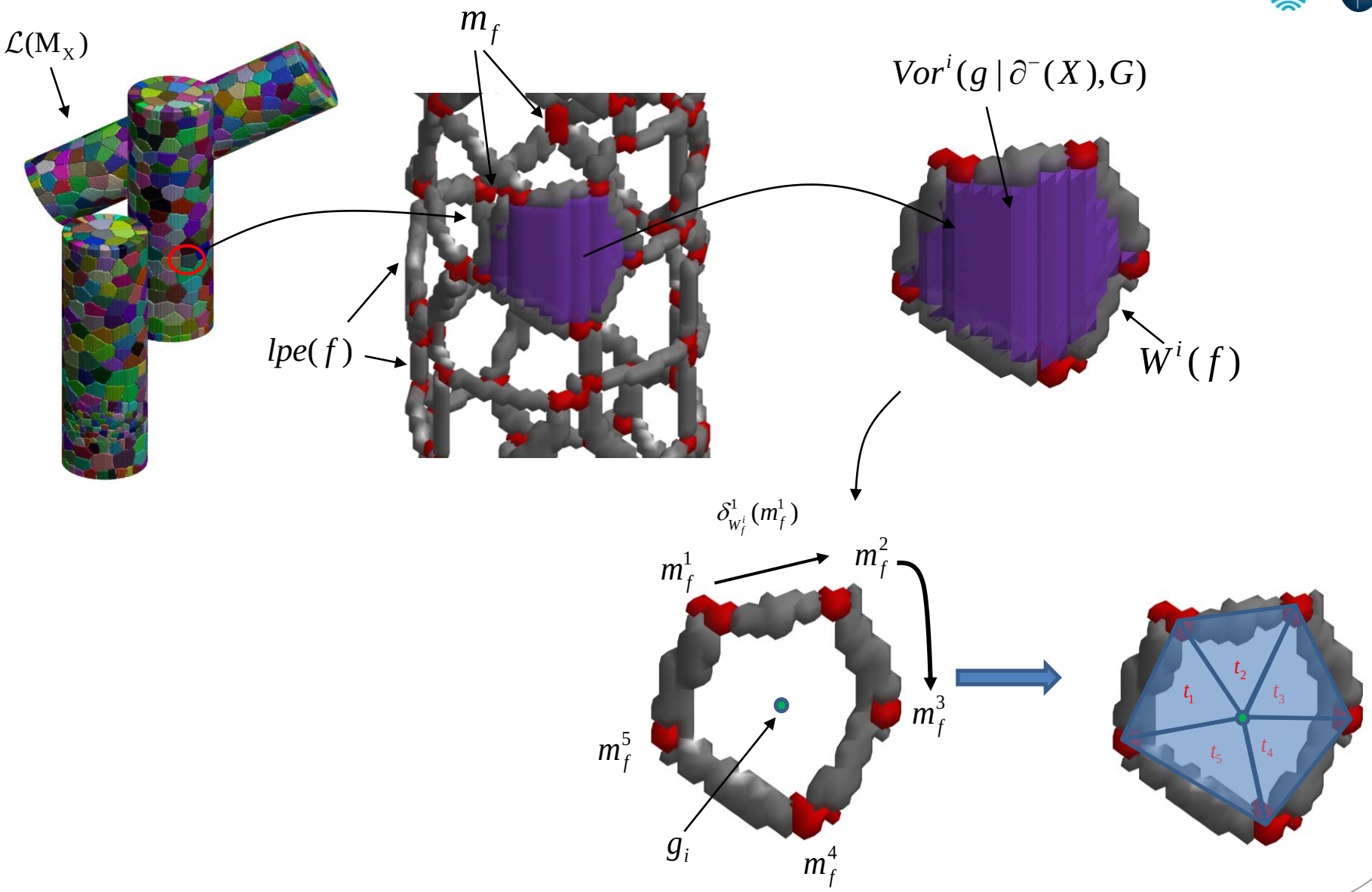


M_X

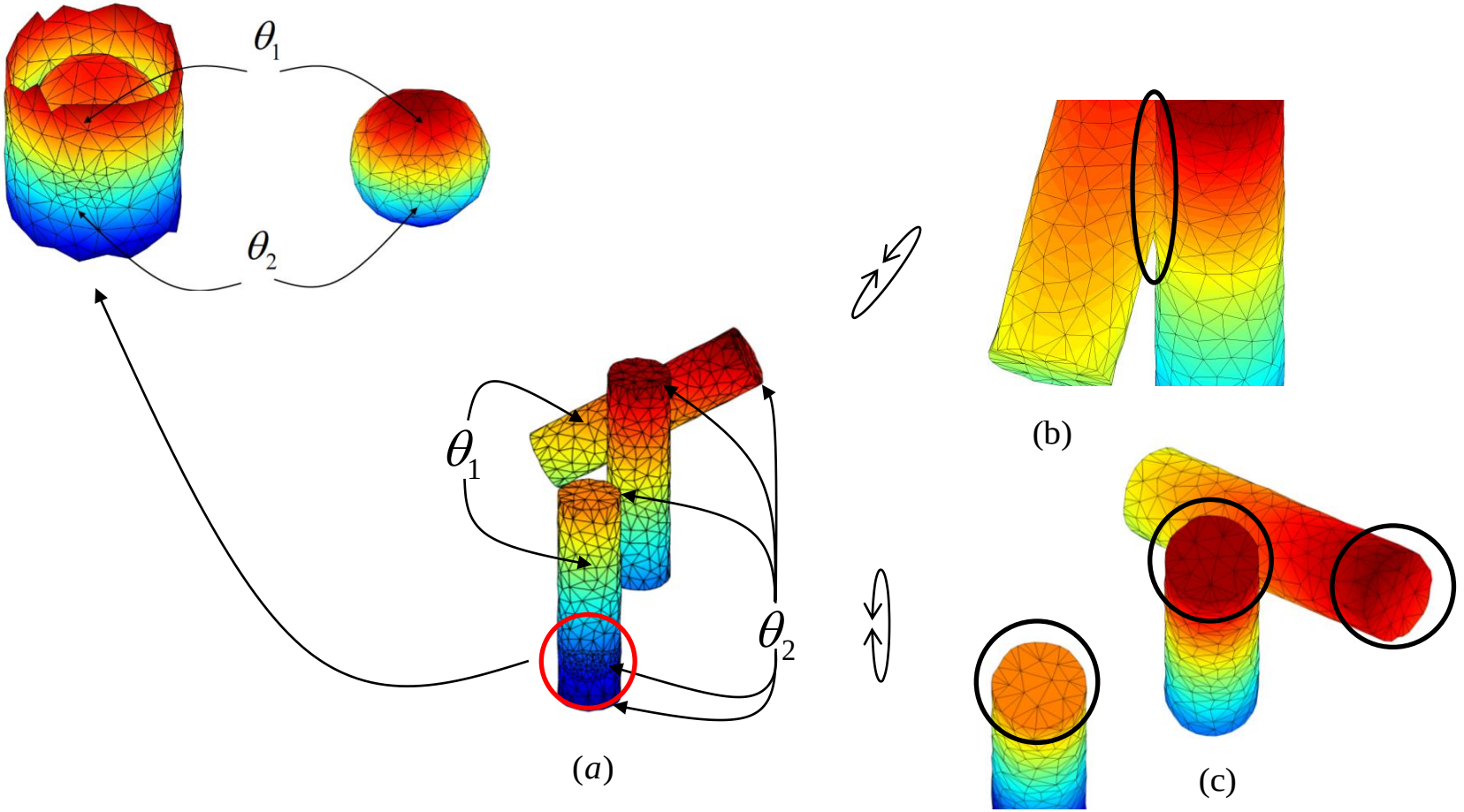


$\mathcal{L}(\mathrm{M}_X)$

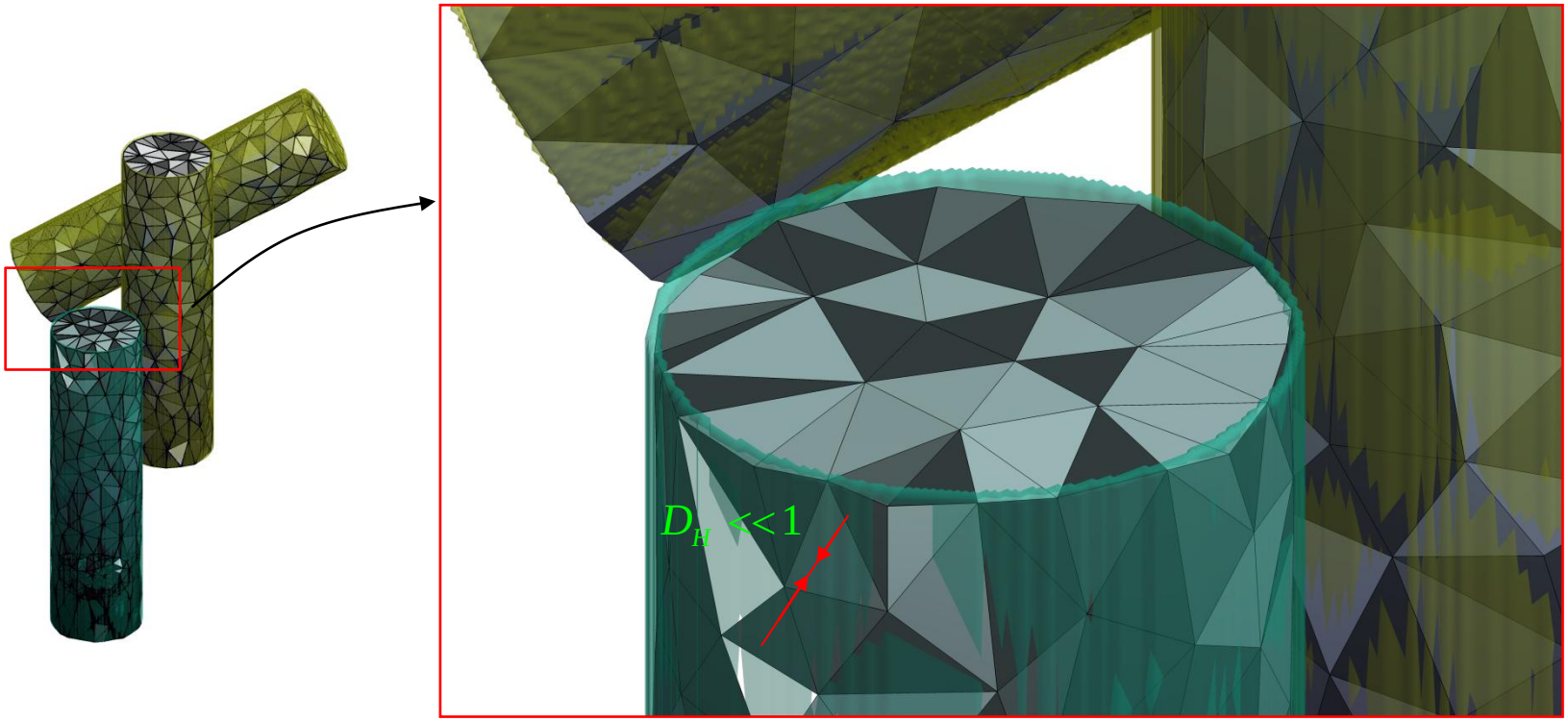
Graph-M


$\mathcal{L}(M_X)$
m_f
$Vor^i(g \mid \partial^-(X), G)$
$lpe(f)$
$W^i(f)$
$\delta^1_{W_f^i}(m_f^1)$
m_f^1
m_f^2
m_f^3
m_f^4
m_f^5
g_i
t_1
t_2
t_3
t_4
t_5

Mesh



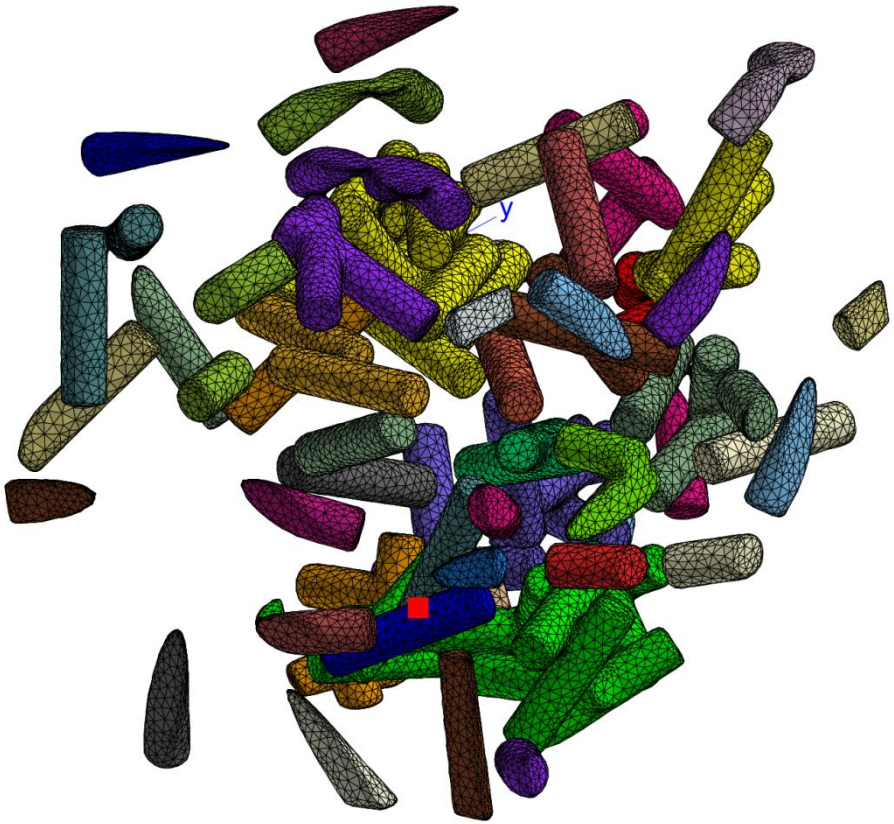
Hausdorff distance



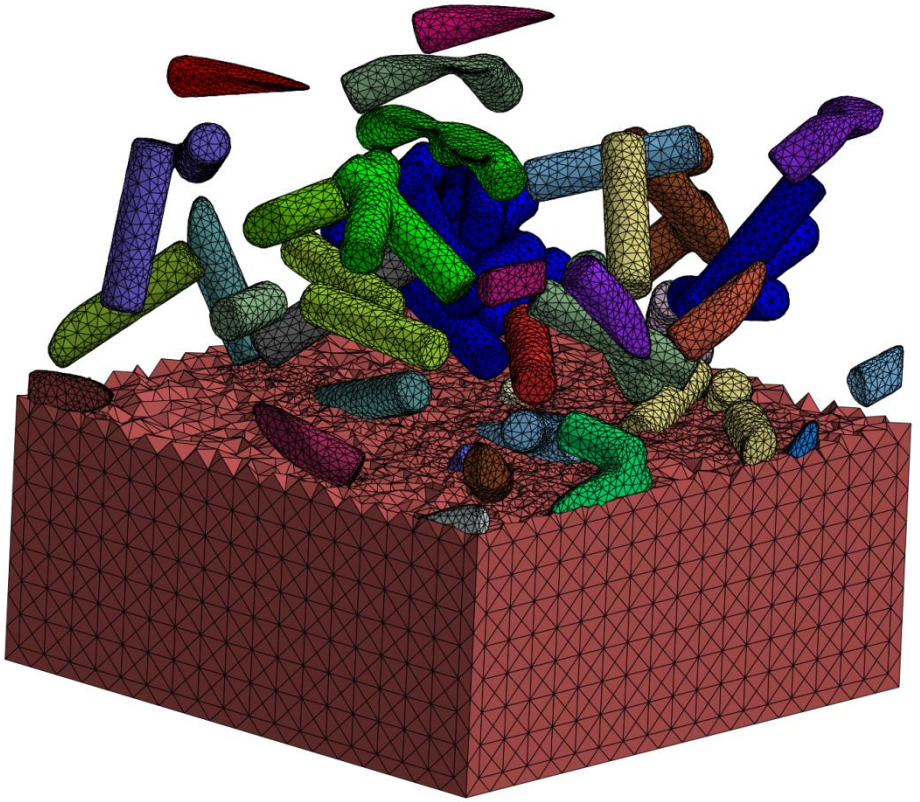
$$d(\partial^-(X), \mathrm{T}(X)) = \max\left\{ \sup_{x \in \partial^-(X)} d(x, \mathrm{T}(X)), \sup_{x \in T(X)} d(x, \partial^-(X)) \right\}$$

$$d(\partial^-(X), \mathrm{T}(X)) = \inf\left\{ \varepsilon, \partial^-(X) \subset \delta_{B(\varepsilon)}(\mathrm{T}(X)), T(X) \subset \delta_{B(\varepsilon)}(\partial^-(X)) \right\}$$

Boolean scheme

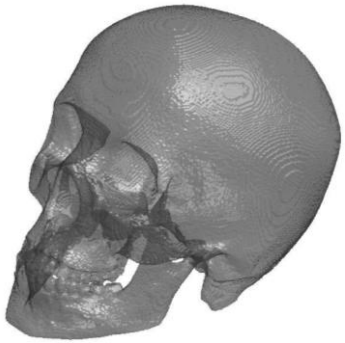
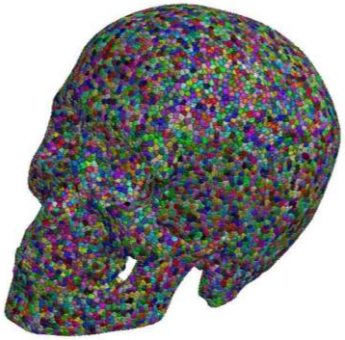
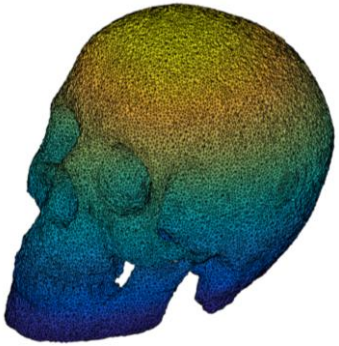
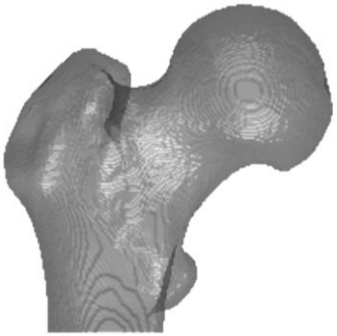
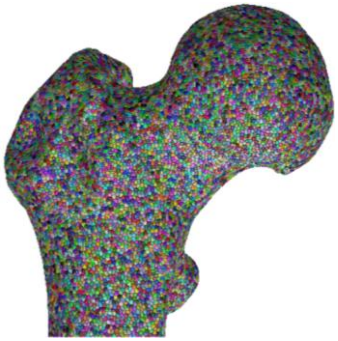
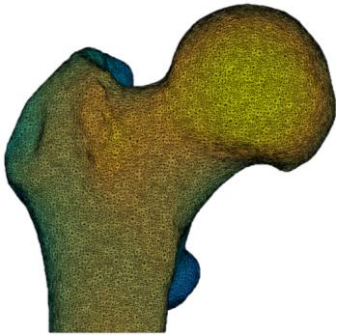


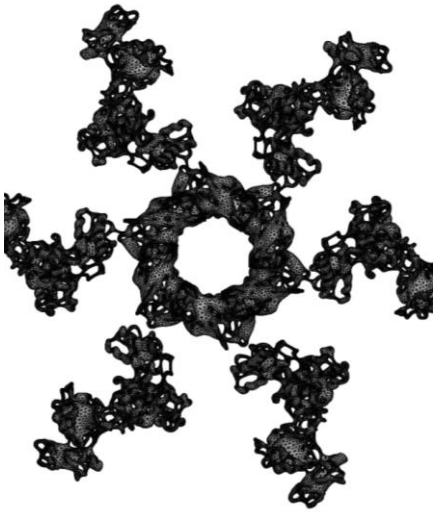
X

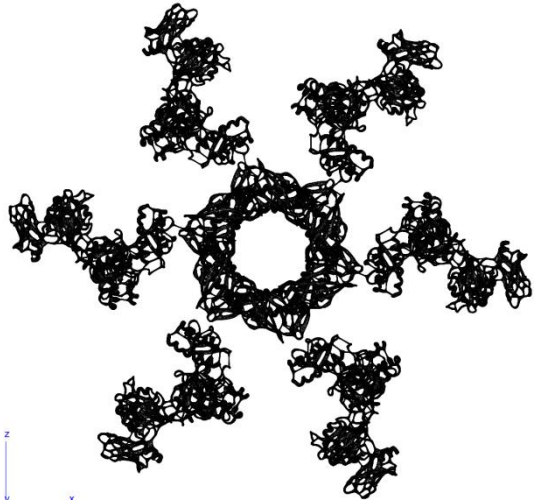


$X \cup X^c$

Other results

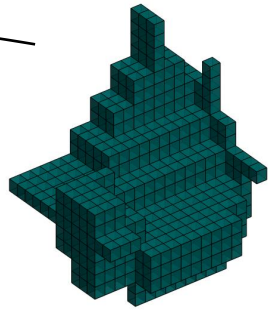
Voxelized representation of an image	Geodesic Voronoï	Morphological triangulation associate
		
		



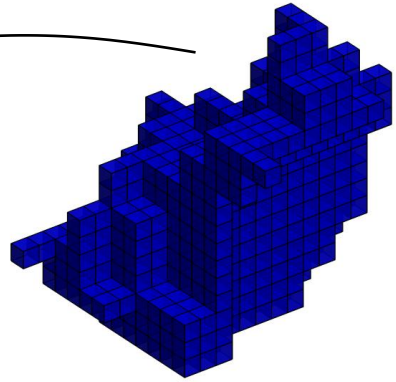


Labelled 3D image

Non convex phase morpholoy

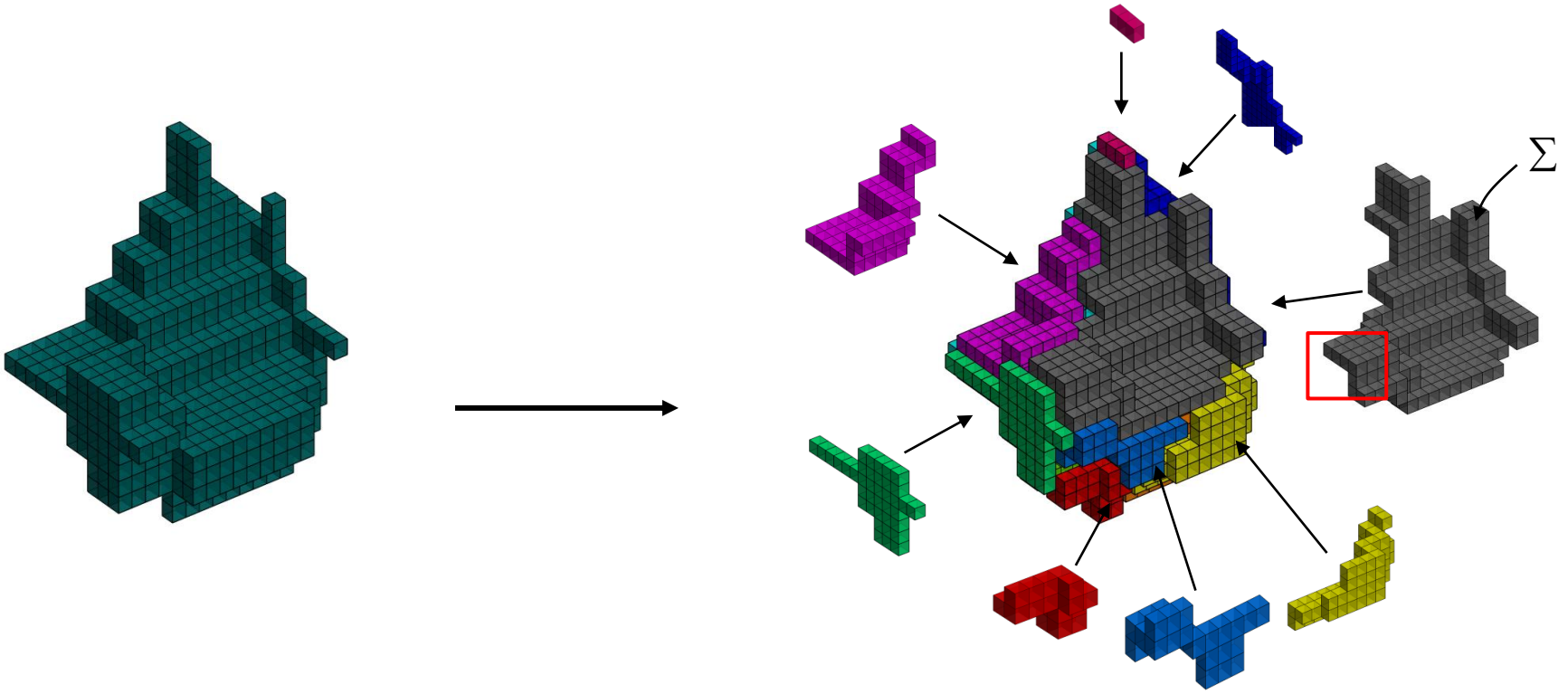


Single connected-component morphology



Phase property

Adjacency of contact surfaces between labels



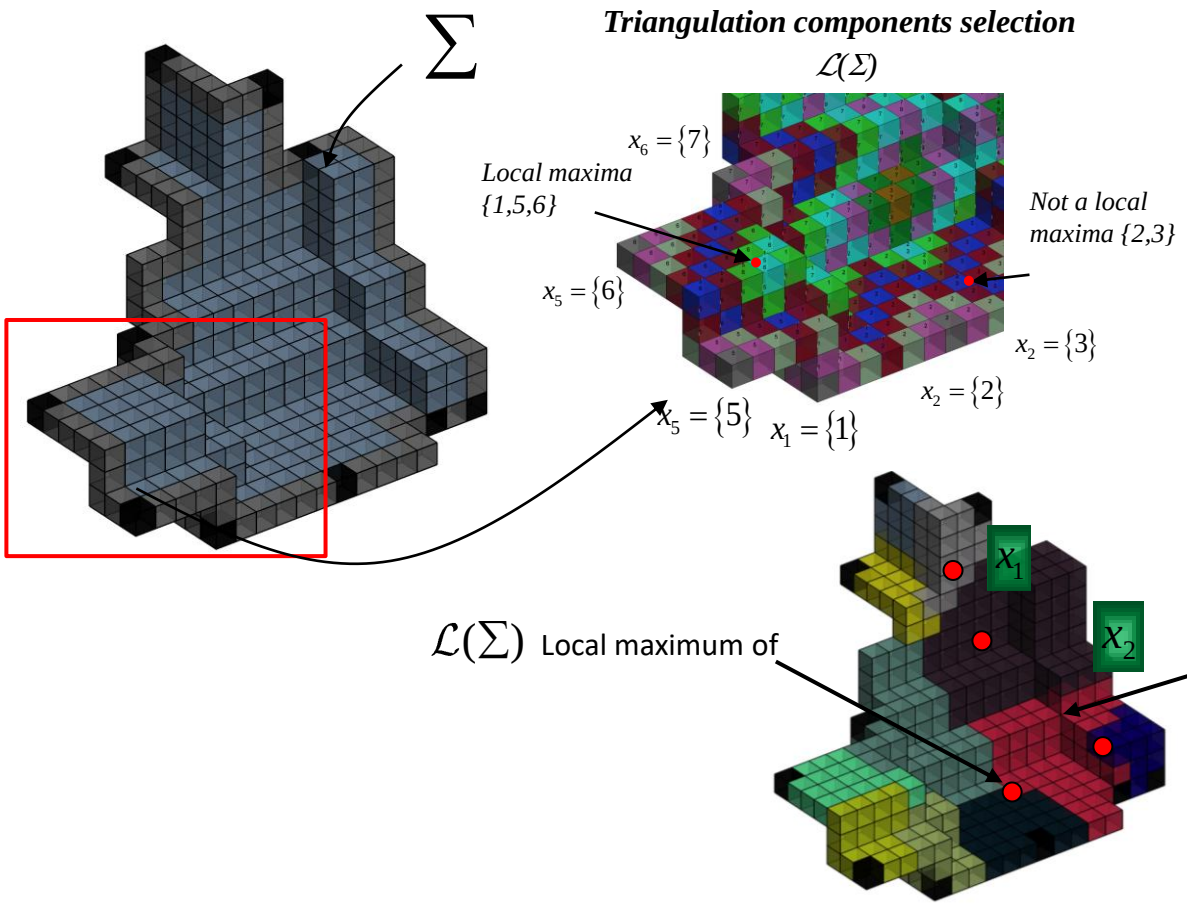
$$g^{+}(f) = \delta_B(f) - f$$

External Beucher gradient

Geodesic surface triangulation

3D Geodesic open surface Voronoi consideration

Ensemblist geodesic M-Graph defined on an open surface



Ensemblist labelisation

$$\mathcal{L} : \Sigma \to \mathcal{P}(\{1, \ldots, m\})$$

Ensemblist distance function

$$\forall y \in \Sigma, \delta(y) = \min_{1<i<m} d_6^{\Sigma}(y, x_i)$$

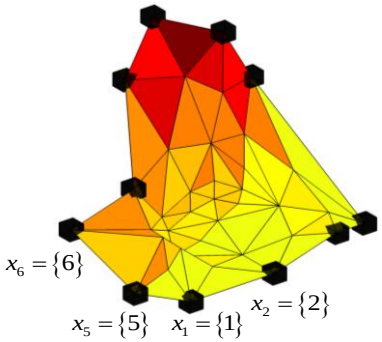
$$\begin{cases} \delta(y) = d_6^{\Sigma}(y, x_{i1}) = \ldots = d_6^{\Sigma}(y, x_{ip}) \\ \forall k \notin \{i_1, \ldots i_p\}, d_6^{\Sigma}(y, x_k) > \delta(y) \end{cases}$$

whith d_6^{Σ} the d_6 – geodesic distance in $\in \Sigma$

Geodesic Local maxima detection

$$L \overset{def}{=} \{i_1, \ldots i_p\} \in \mathcal{L}(\Sigma), L \textbf{ maximal in } \mathcal{L}(\Sigma)$$

Iterativ construction

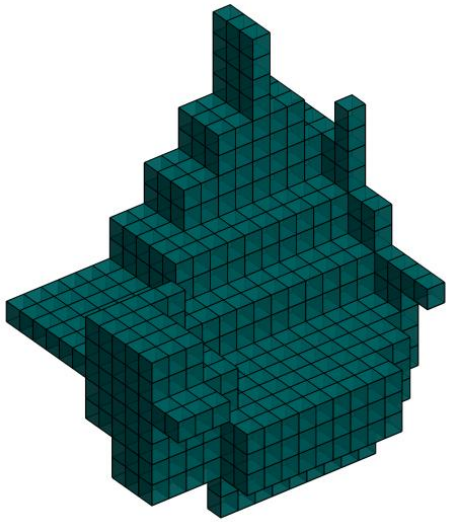


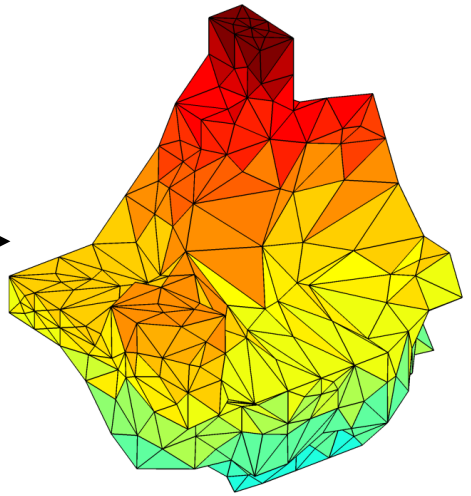
Geodesic surface triangulation :

Vertices : $X_1, \ldots, X_m$

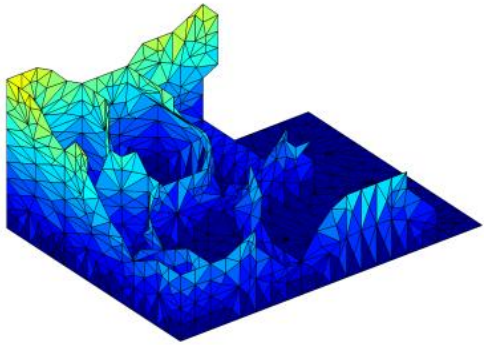
and

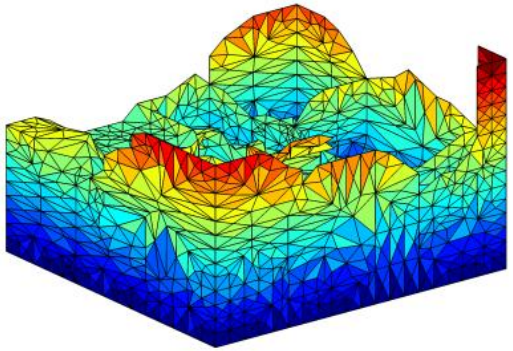
Edges : $[x_i, y_L]$ where $i_j \in L$





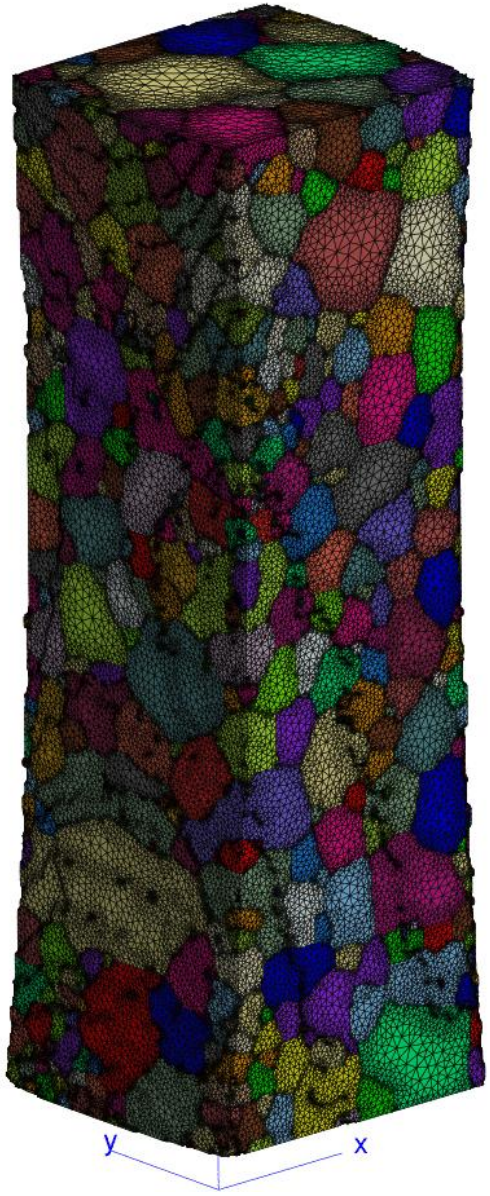
Phase triangulation





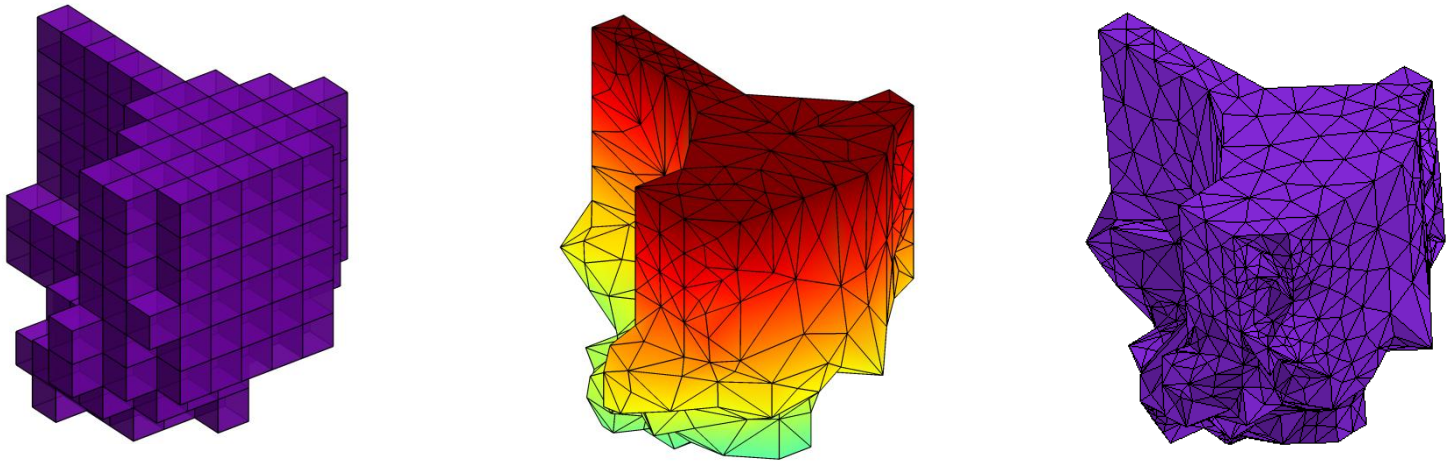
Mesh

3D generalized M-Graph on a labelled image

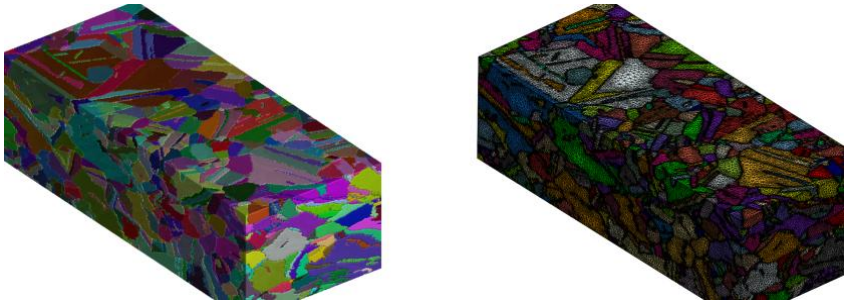


1368776 nodes

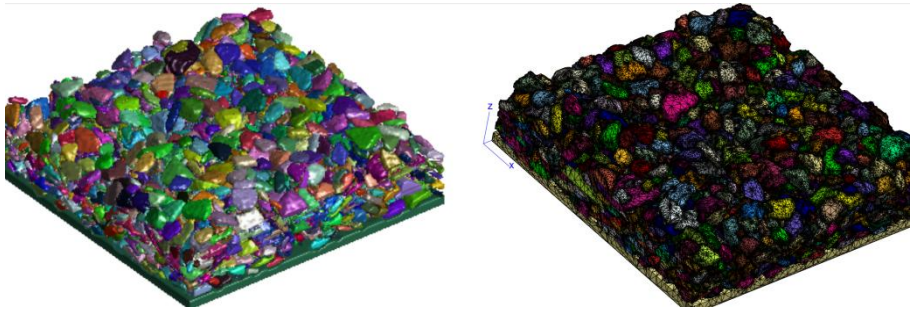
Adaptative mesh



Other realisations



Non convex shape metallic material



Simulation of cold spray coating