

# Topic 2 – Theory about X-ray attenuation: The Beer-Lambert Law

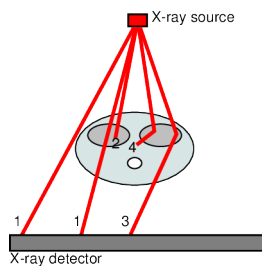
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## X-photons/matter Interactions (1/2)

- X-photons cross matter;
- During their path into any material, they can interact with matter.



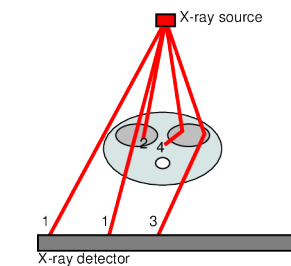
*Illustration of X-ray photon/matter interaction*

1. Directly transmitted photons (no interaction);
2. Absorbed photons;
3. Scattered photons;
4. Absorbed scattered photons.

## X-photons/matter Interactions (2/2)

**For most X-rays imaging modalities, only directly transmitted photons are essential:**

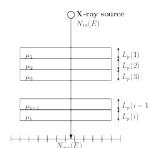
- Scattered photons decrease the image quality;
- Absorbed photons do not reach the detector;
- Scattered photons may be ignored (but not necessarily).



*Illustration of X-ray photon/matter interaction*

1. Directly transmitted photons (no interaction);
2. Absorbed photons;
3. Scattered photons;
4. Absorbed scattered photons.

## Beer-Lambert Law (Attenuation Law)



*Illustration of the Beer-Lambert law*

- $N_{out}(E) = N_{in}(E) e^{-\sum_i \mu_i(E, Z) L_p(i)}$ 
  - $N_{in}(E)$  the number of incident photons at energy  $E$ ;
  - $N_{out}(E)$  the number of transmitted photons of energy  $E$ ;
  - $\mu_i$  the linear attenuation coefficient (in  $\text{cm}^{-1}$ ) of the  $i$ th object. It depends on:
    - ★  $E$  the energy of incident photons;
    - ★ the material density of the object;
    - ★  $Z$  the atomic number of the object material.
  - $L_p(i)$  the path length of the ray in the  $i$ th object.
- $E_{out} = N_{out}(E) \times E$ 
  - $E_{out}$  the energy received by the pixel, i.e. as recorded in the X-ray image.

## Example (monochromatic case)

[http://gvirtualxray.sourceforge.net/validation/validation\\_03/beer\\_lambert\\_law\\_monochromatic.php](http://gvirtualxray.sourceforge.net/validation/validation_03/beer_lambert_law_monochromatic.php)

## Beer-Lambert Law in the polychromatic case

- There are more than one energy in the incident beam spectrum
- Just iterate over the energy channels:

$$E_{out} = \sum_j E_j \times N_{out}(E_j)$$

$$E_{out} = \sum_j E_j \times N_{in}(E_j) e^{(-\sum_i \mu_i(E_j, Z) L_p(i))}$$

with j the j-th energy channel

## Example (polychromatic case)

[http://gvirtualxray.sourceforge.net/validation/validation\\_05/beer\\_lambert\\_law\\_polychromatic.php](http://gvirtualxray.sourceforge.net/validation/validation_05/beer_lambert_law_polychromatic.php)

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