

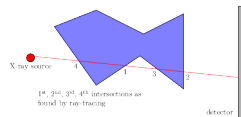
# Topic 3 – Implementing the Beer-Lambert Law on GPU using OpenGL

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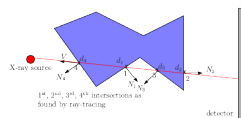
## Path Length: Naive Approach



*Is finding intersections in the right order important?*

1. Detect every intersection between a ray and the objects;
2. Sort intersection (Can be handled by GPUs using depth-peeling, a multi-pass rendering technique for semi-transparent polygonal objects without sorting polygons);
3. Compute path length.

## Path Length: L-Buffer



*Finding intersections in any order does not matter*

### Intersection sorting is actually not needed!

- By convention normals are outward;
- A ray penetrates into an object when the dot product between the view vector ( $V$ ) and the normal ( $N_i$ ) at the intersection point is positive;
- It leaves an object when the dot product is negative.

## L-Buffer Implementation

$$L_p = \sum_i -\text{sgn}(\mathbf{V} \cdot \mathbf{N}_i) \times d_i$$

- $i$  refers to  $i$ th intersection in an arbitrary order;
- $d_i$  distance from X-ray source to intersection point;
- $\text{sgn}(\mathbf{V} \cdot \mathbf{N}_i)$  stands for the sign of the dot product between  $\mathbf{V}$  and  $\mathbf{N}_i$ ;
- In a shader program, compute:
  - $\text{sgn}(\mathbf{V} \cdot \mathbf{N}_i)$ ;
  - $d_i$  the distance between the X-ray source and the intersection;
  - Assign  $-\text{sgn}(\mathbf{V} \cdot \mathbf{N}_i) \times d_i$  as the fragment value.
- For each pixel, compute  $L_p$  thanks to high-dynamic range and OpenGL blending function (pixel values may not be between 0 and 1).

See <http://dx.doi.org/10.2312/LocalChapterEvents/TPCG/TPCG09/025-032> for more details.

## Multipass Rendering Pipeline

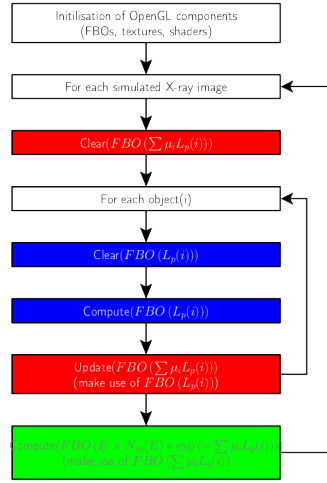
$$\text{pixel} = E \times N_{\text{out}}$$

$$\text{pixel} = E \times N_{\text{in}}(E) e^{-\sum_i L_p(i)}$$

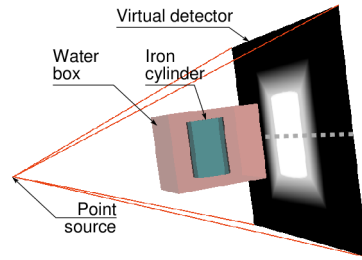
- Needs 3 FBOs with high-dynamic range capability for off-line rendering;
- For each object of the scene:
  1. Compute  $L_p(i)$ ;
  2. Update results of  $\sum_i L_p(i)$ .
- For the final image only:
  1. Compute  $N_{\text{out}}$ ;
  2. (Optional when direct display only is needed).

## Adding the Beam Spectrum

- Take into the different energies within the incident beam;
- This is known as *beam hardening*;
- Iterate over several energy channels:
  - $\text{pixel} = \sum_j E_j \times N_{\text{out}}(E_j)$



*OpenGL pipeline to compute the Beer-Lambert law (monochromatic case).*



*Simulation parameters*

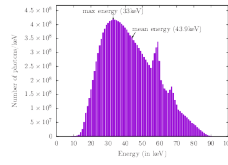
- $\text{pixel} = \sum_j E_j \times N_{\text{in}}(E_j) e^{-\sum_i \mu(E_j, Z) d_i}$
- Example:

## Simulation with Different Source Shapes

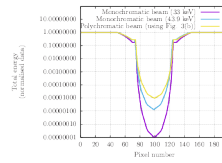
- Take into account the focal spot of the X-ray source;
- Iterate over several point sources within the volume of the focal spot:
  - $\text{pixel} = \sum_k \sum_j E_j \times N_{\text{in}}(E_j) e^{-\sum_i \mu(E_j, Z) d_i(k)}$
  - See blur in the corresponding image below.

## Final Simulation Flowchart

- Iterate over several energy channels: Three extra for loops;
- Iterate over several point sources within the volume of the focal spot: One extra for loop.



*Polychromatic beam spectrum for 90kV  
X-ray tube peak voltage*



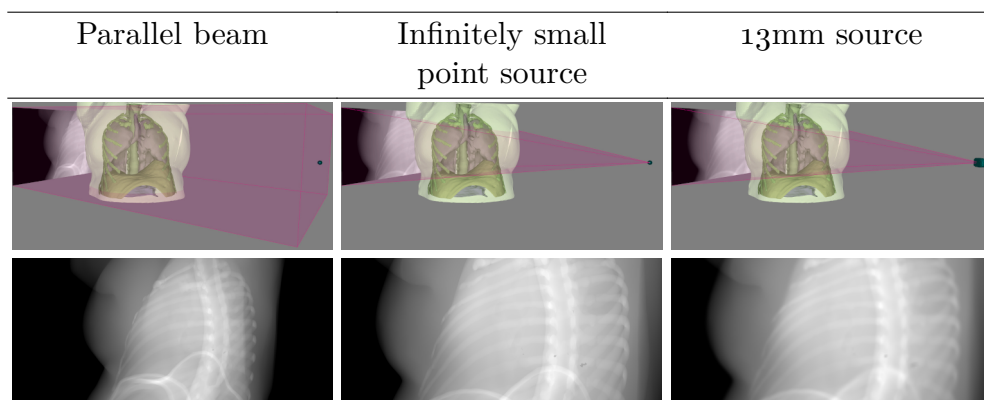
*Intensity profiles, see  
dash line in image above*

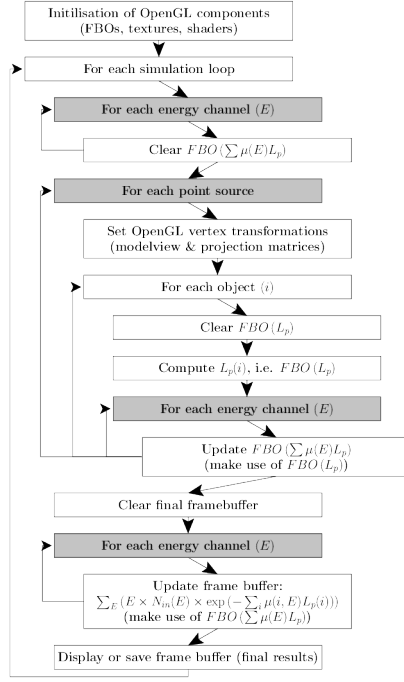
## Bibliography (links)

- DOI: 10.2312/LocalChapterEvents/TPCG/TPCG09/025-032
- DOI: 10.1007/s11548-009-0367-1
- DOI: 10.2312/egp.20101026
- DOI: 10.1016/j.compmedimag.2015.12.002

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*Final OpenGL pipeline*