An explicit finite element method for Navier-Stokes-Brinkman equations

LIANG YANG

Image based simulation for industry 2021 (IBSim-4i)

October 2021, London

Research Interest

Career

- Lecturer in **Marine Renewable Energy Systems** at Cranfield University
- Postdoc in Computational Physics, MSc and PhD in Computational Mechanics

➢ Finite element analysis and massively parallel solvers
➢ Coupling with radiation transport

➢ Multiphase fluid-rigid-flexible structure interaction
➢ Wave-structure interaction

➢ Wave tank experiment

Twitter: lyang200809

Linkedin: https://www.linkedin.com/in/lyang7/

https://www.cranfield.ac.uk/people/dr-liang-yang-23276693

A few challenges for image based simulations

Popular algorithm

- lattice Boltzmann method
- finite difference method
- finite volume method
- discontinuous Galerkin
- finite element

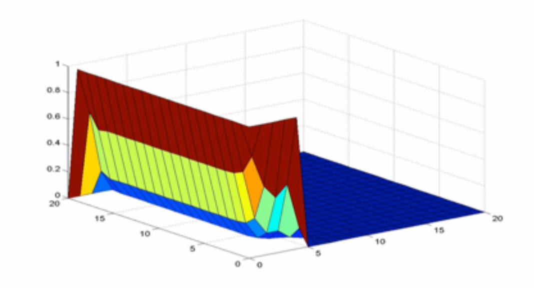
We need

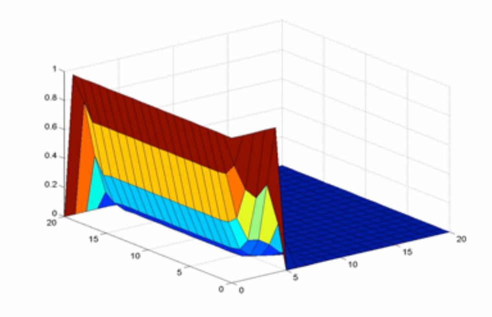
- Extremely large scale voxels
- Memory costing
- Parallel efficiency

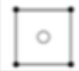
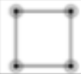
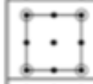
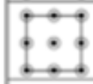
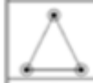
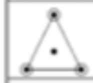
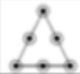
Finite element for fluid flow

Stabilised finite element for

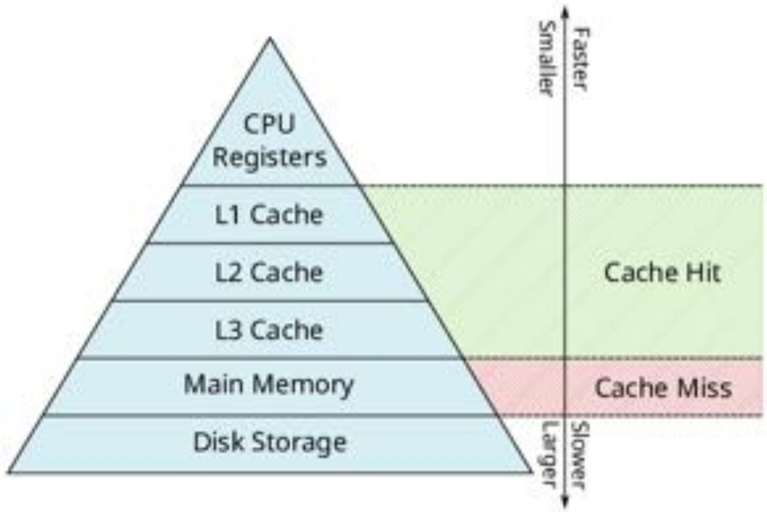
- Convection dominated problem
- Equal order interpolation
- Turbulence modelling
- Matrix-free implementation
- Fully explicit with artificial compressibility

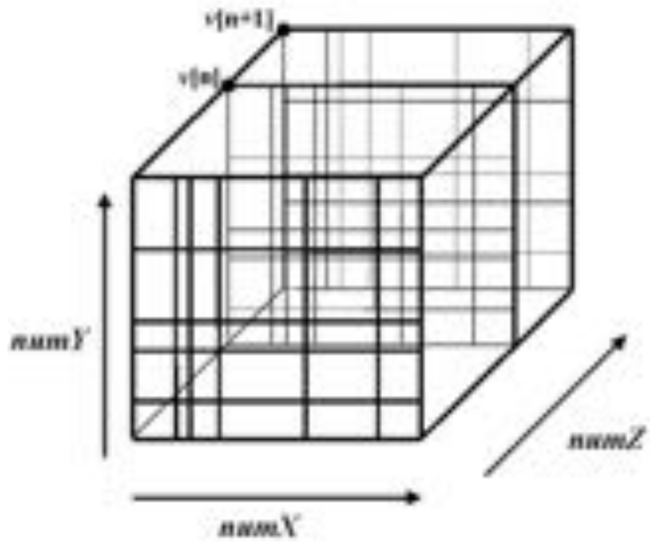




Mixed elements			
Figure	Name	Velocity interpolation	Pressure interpolation
	Q1Q0	Linear	Constant
	Q1Q1	Linear	Linear
	Q2Q1	Quadratic	Linear
	Q2Q2	Quadratic	Quadratic
	P1P0	Linear	Constant
	P1P1	Linear	Linear
	Mini	Linear	Linear
	P2P1	Quadratic	Linear
	P2P2	Quadratic	Quadratic
• Velocity node ○ Pressure node			

Large scale computing





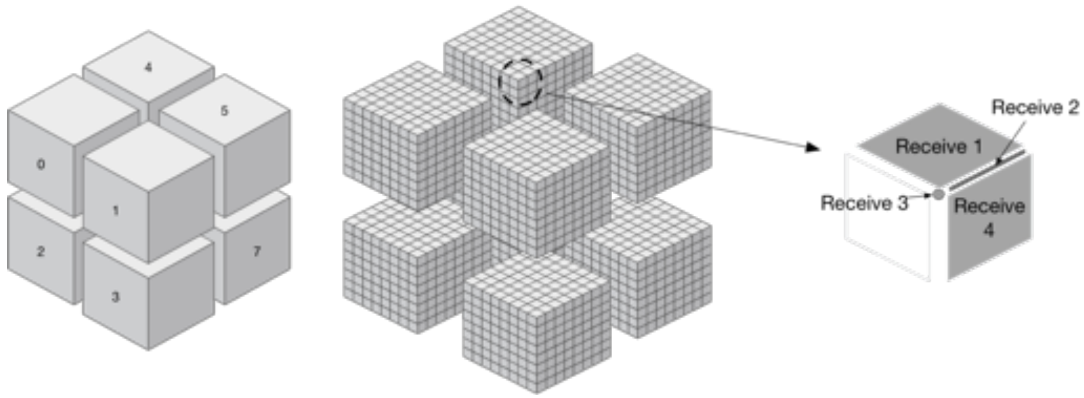
- Matrix-Free scheme
- avoid matrix-assembly and simplify domain decomposition

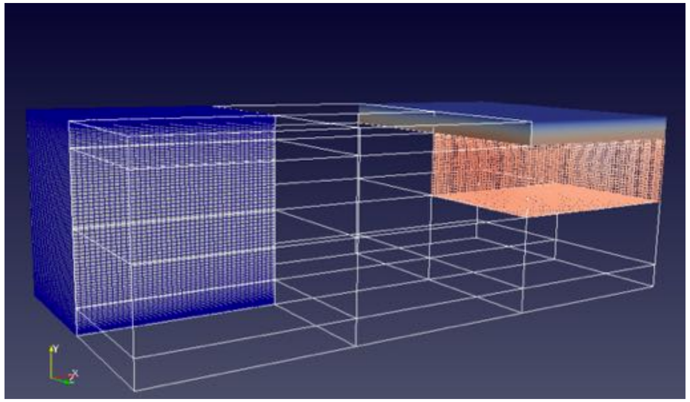
- Cartesian grid
- avoid indirect addressing

PyEFEM

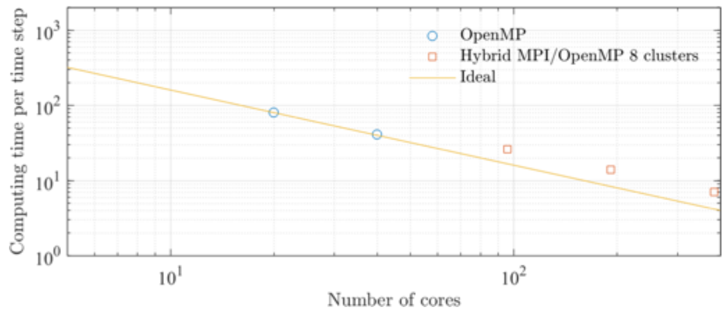
- Governing equations Navier-Stokes equations of fluid dynamics; any advection-diffusion-reaction problem, Naiver-Stokes-Brinkmann
- Dimensionality: 2D and 3D
- External routines: Python, Numpy, MPI
- Element types: Quadrilaterals, Hexahedra
- Platforms: Tier 2 cluster
- Spatial Discretisation: Stabilised Finite Element
- Parallelisation: Hybrid OpenMP/MPI
- Build in LES model
- Temporal Discretisation: Explicit
- Solution Files Exported: VTK, Tecplot
- Applications: aerodynamics, aeroacoustics, environmental flow, porous medium flow, haemodynamics.

Scalability and memory usage



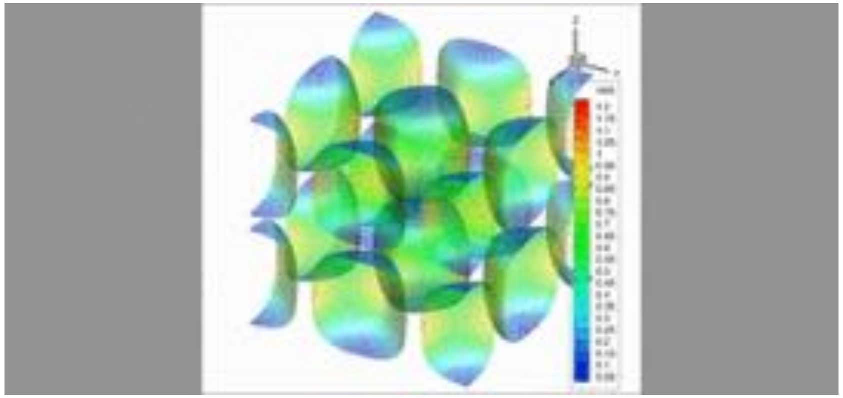


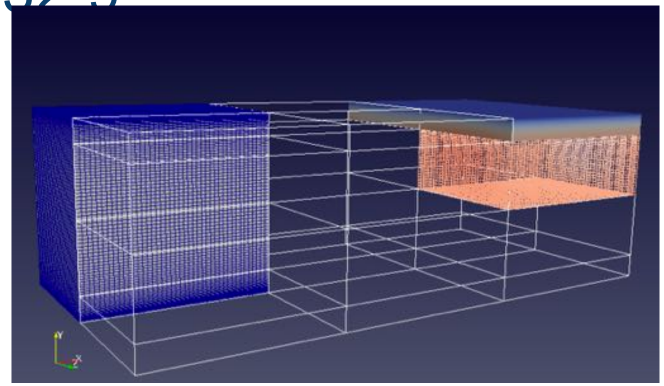
Elements size	Peak memory usage (GB)
128x128x128	0.68
256x256x256	5.44
512x512x512	43.51
1024x1204x1024	351

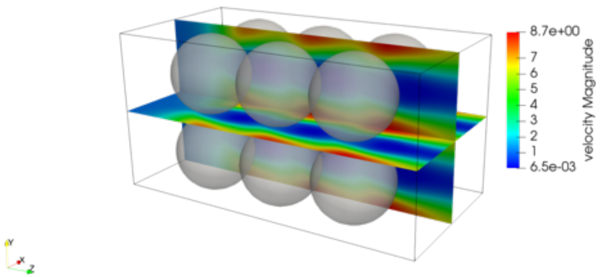


Turbulent flow: LES/DNS

https://doi.org/10.1007/s00466-016-1332-9

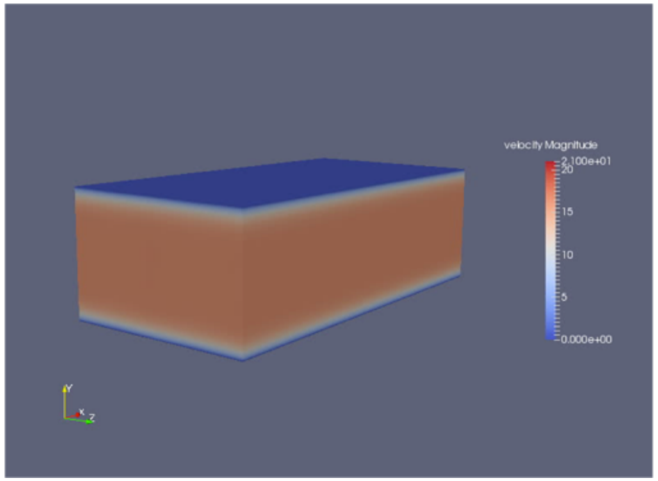






Flow over packed bed

Trailing edge noise



Aeroacoustics applications (unpublished)

Non-reflection boundary conditions

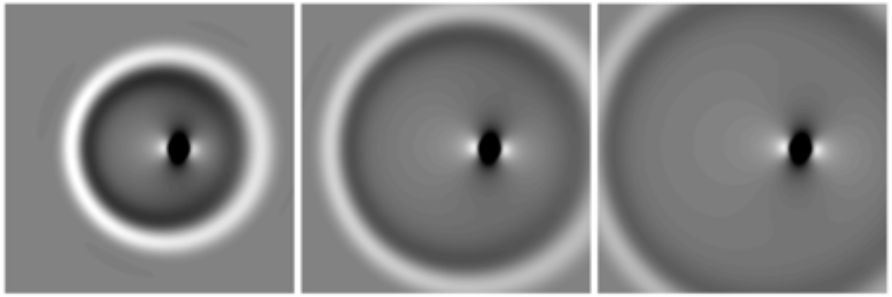


Figure 3: Contours of p_h in the large domain for the big/small domain benchmark problem in 2D. From the left to right: $t = 10$, $t = 15$, $t = 20$, $dx = 0.01$.

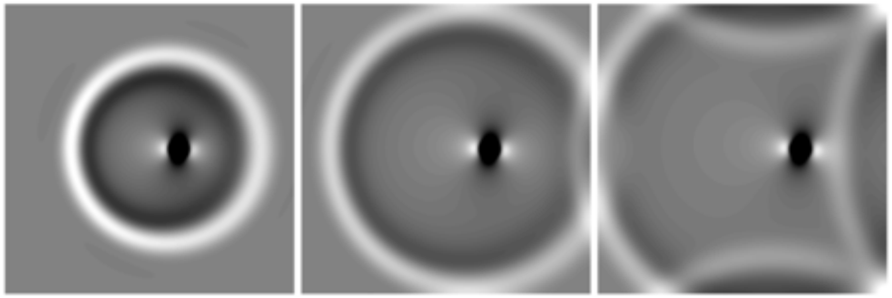
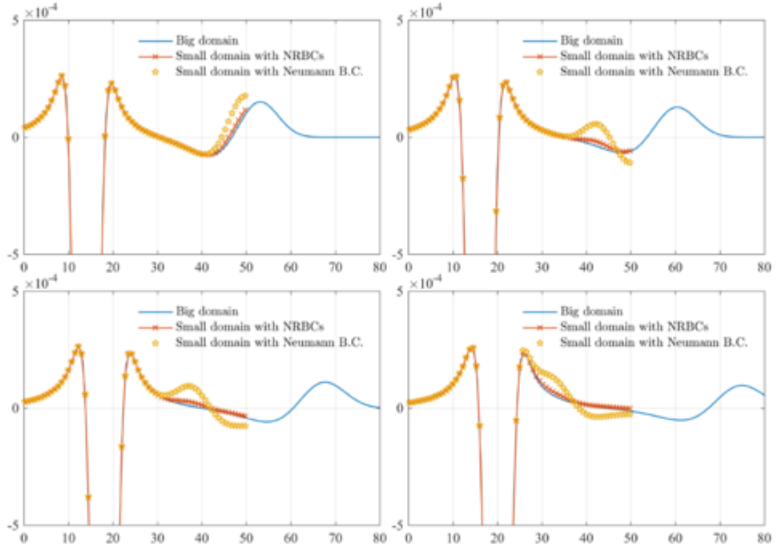
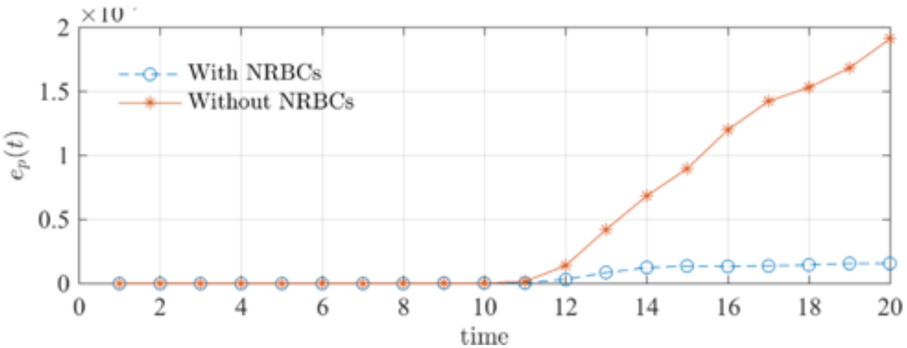


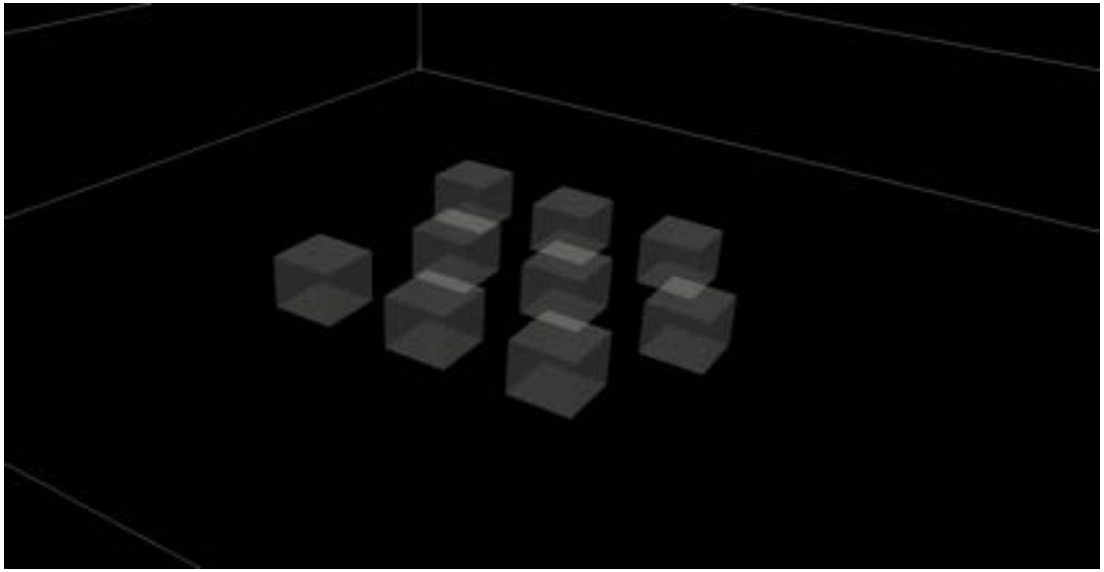
Figure 4: Contours of p_h in the small domain for the big/small domain benchmark problem in 2D. From the left to right: $t = 10$, $t = 15$, $t = 20$, $dx = 0.01$.

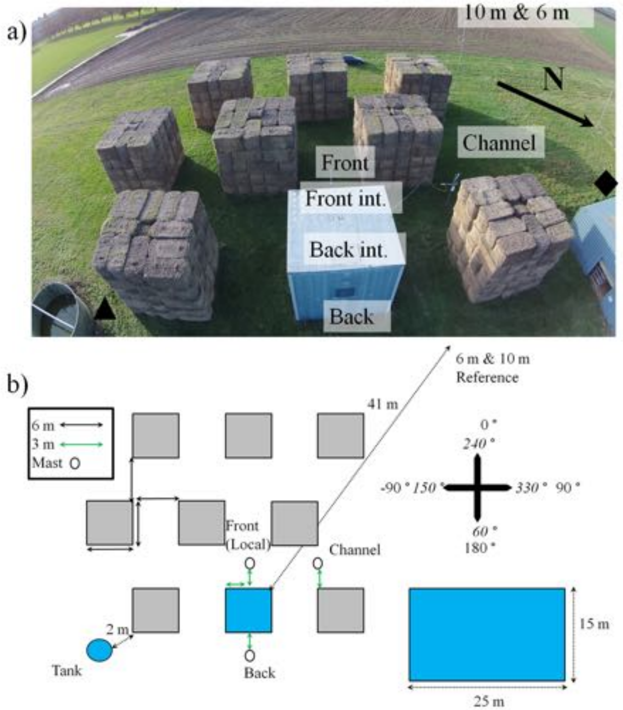




Urban air flow (unpublished)

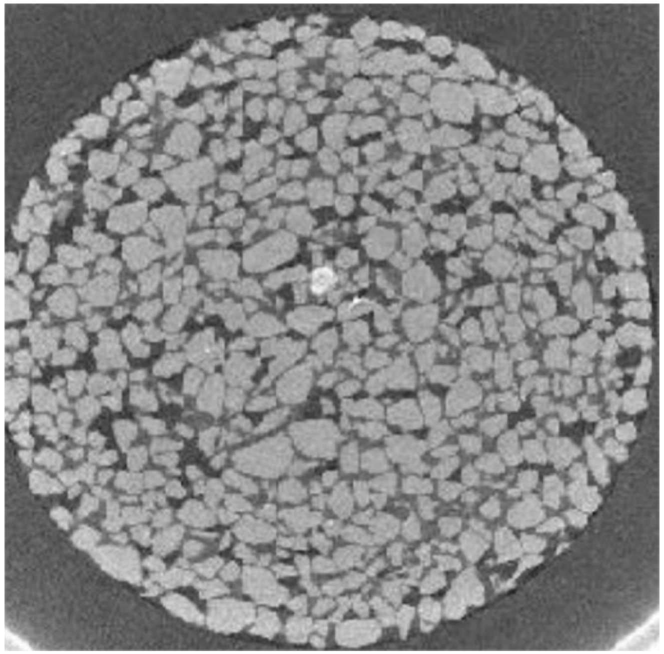
Investigation the influence of a staggered array on the Silsoe cube façade pressures



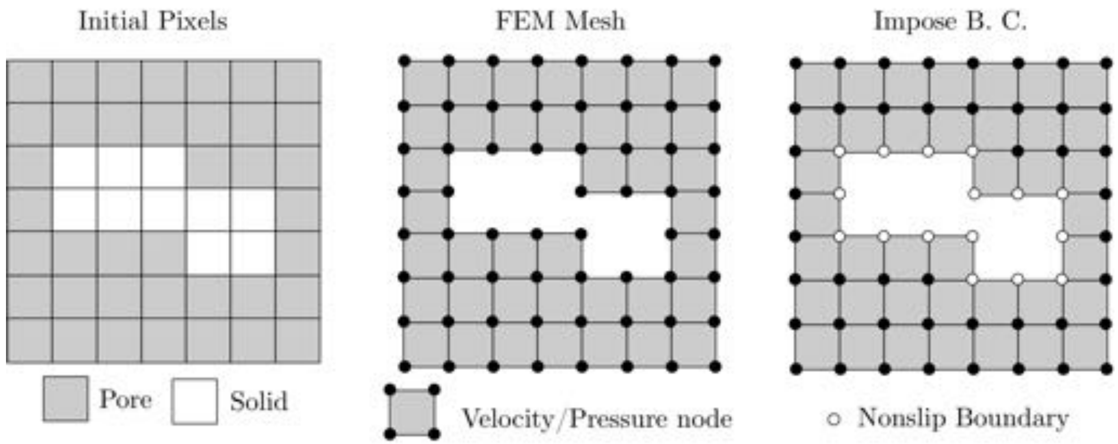


Digital rock physics
https://doi.org/10.1007/s10596-019-09837-4

Permeability study

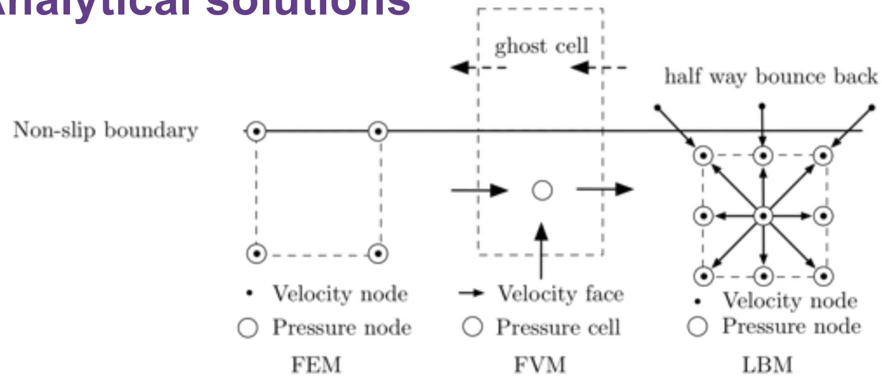


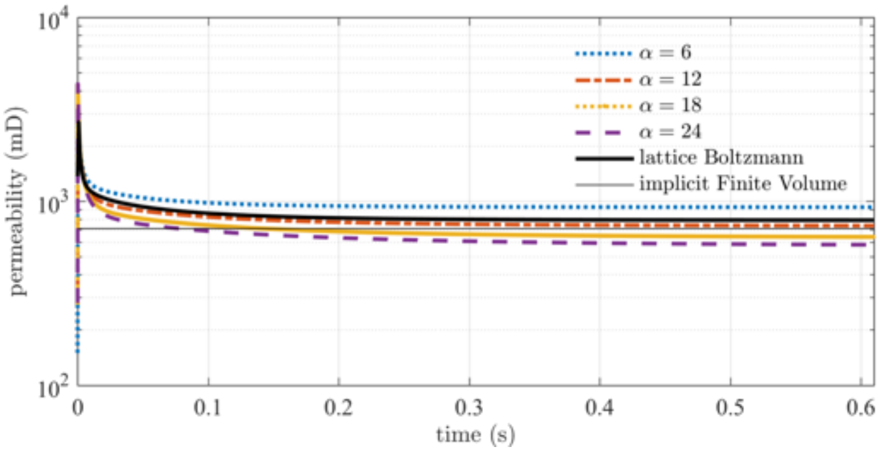
Cross-section of Micro CT Image of Sand Pack LV60A

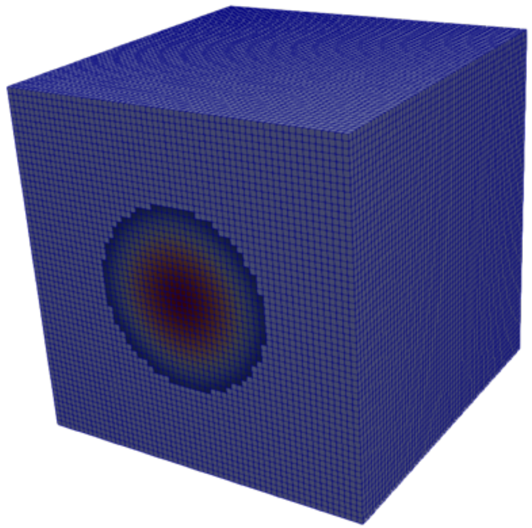


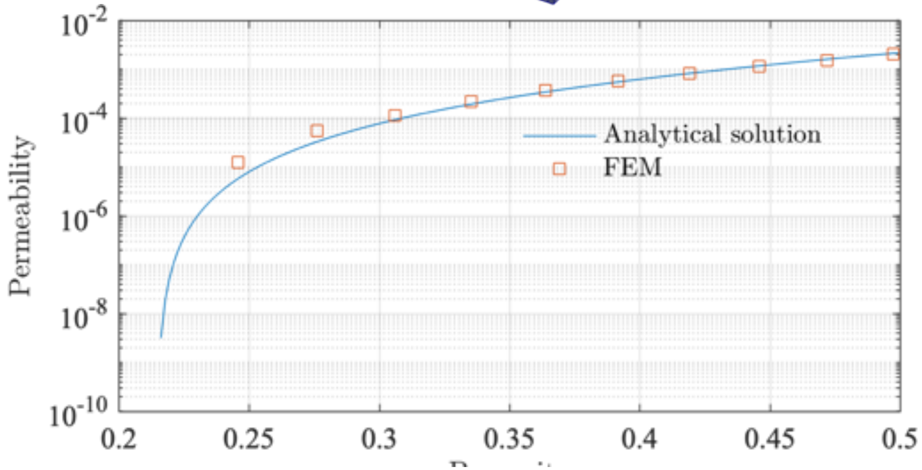
Boundary conditions

Analytical solutions



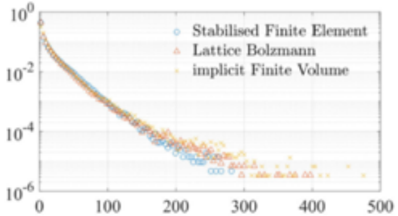




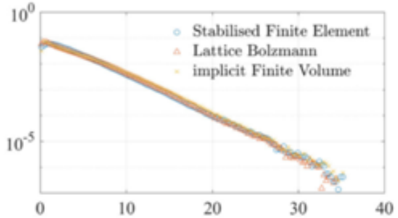


Comparison of different approaches

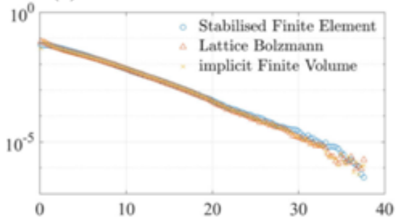
Velocity distribution



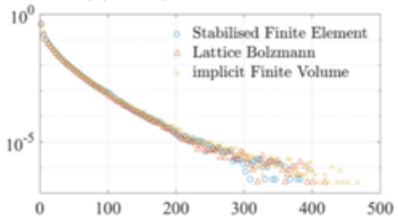
(a) Fontainebleau sandstone 131^3



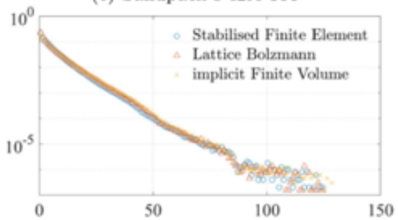
(b) Sandpack LV60A 300^3



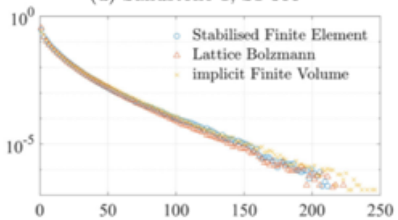
(c) Sandpack F42A 300^3



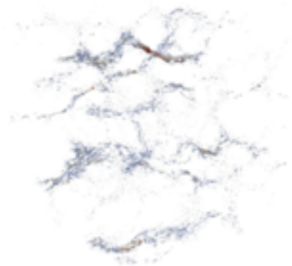
(d) Sandstone 1, S1 300^3

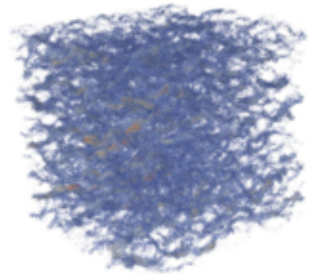


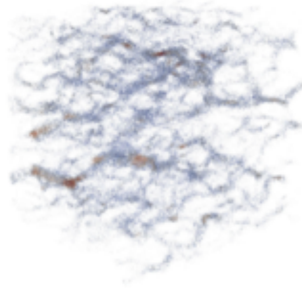
(e) Sandstone 2, S2 300^3

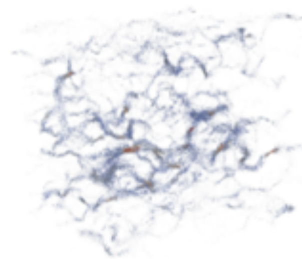


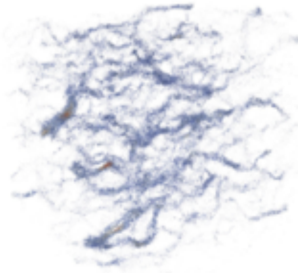
(f) Berea sandstone 400^3

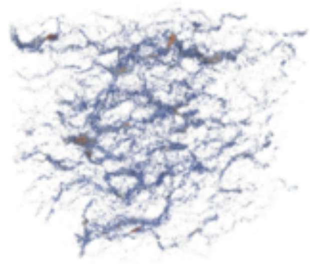

(a) Fontainebleau sandstone 131^3


(b) Sandpack LV60A 300^3


(c) Sandpack F42A 300^3


(d) Sandstone 1, S1 300^3


(e) Sandstone 2,S2 300^3


(f) Berea sandstone 400^3

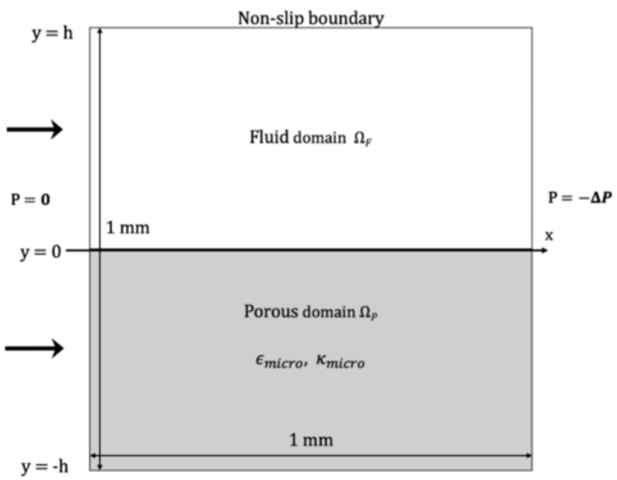
Navier-Stokes-Brinkman equations

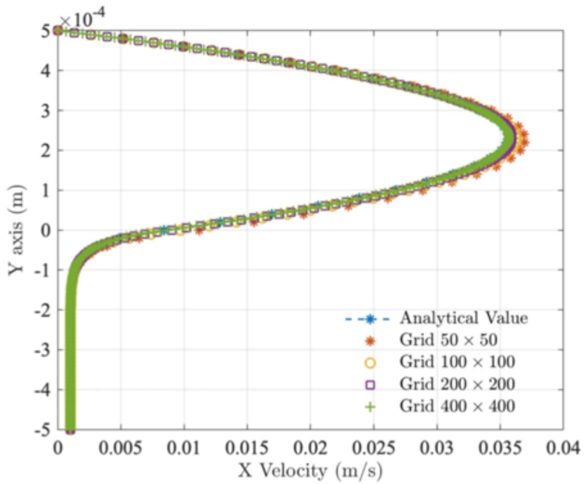
Grayscale rocks

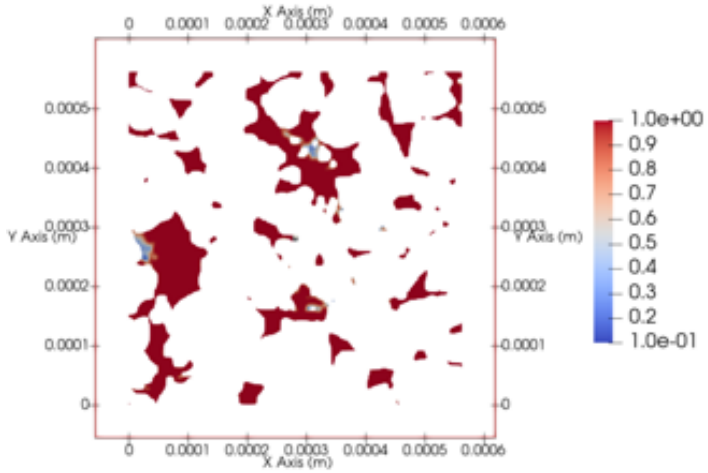
The average Navier-Stokes-Brinkman equations are defined as

$$-\frac{\nu}{\epsilon_{\text{micro}}}\nabla^2\overline{\boldsymbol{u}} + \overline{\boldsymbol{u}}\cdot\boldsymbol{\nabla}\overline{\boldsymbol{u}} + \boldsymbol{\nabla}\overline{p} + \nu\kappa_{\text{micro}}^{-1}\overline{\boldsymbol{u}} = \boldsymbol{f}$$

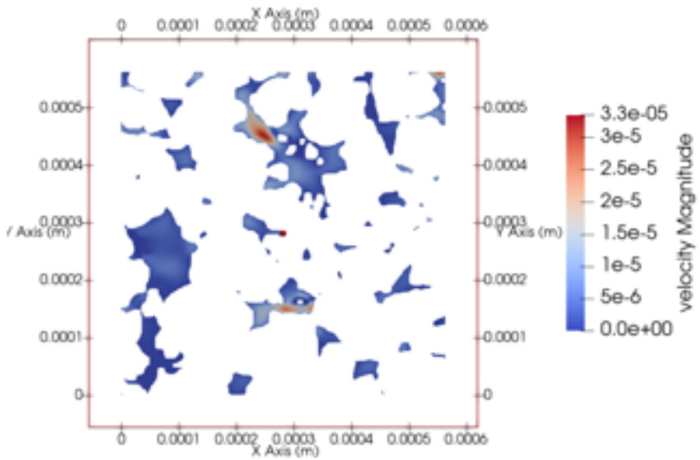
$$\boldsymbol{\nabla}\cdot\overline{\boldsymbol{u}} = 0$$







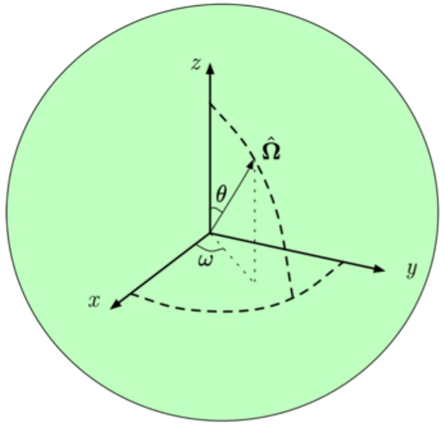
(a) Micro-porosity Distribution



(b) Velocity Distribution

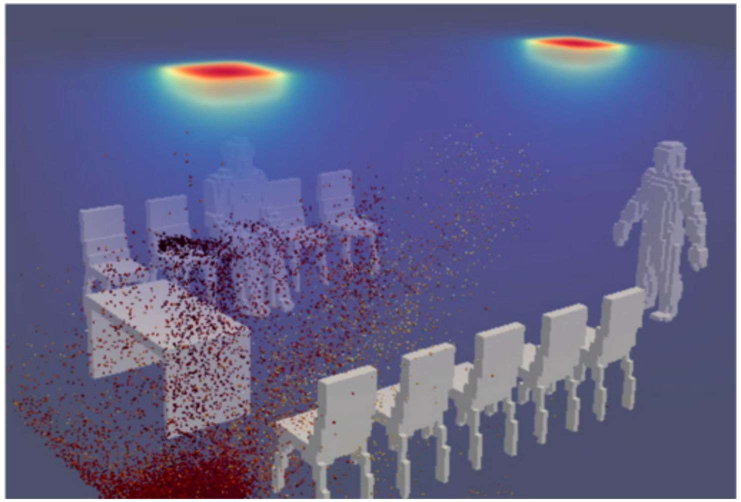
Wyvern: fluid-particle-radiation coupling solver

$$\frac{\partial}{\partial t}\psi\left(\boldsymbol{r},\widehat{\boldsymbol{\Omega}},E,t\right)+\widehat{\boldsymbol{\Omega}}\cdot\nabla\psi\left(\boldsymbol{r},\widehat{\boldsymbol{\Omega}},E,t\right)+\Sigma_t(\boldsymbol{r})\psi\left(\boldsymbol{r},\widehat{\boldsymbol{\Omega}},E,t\right)=\int_{\widehat{\Omega}'}\mathrm{d}\widehat{\boldsymbol{\Omega}}'\Sigma_s\left(\boldsymbol{r},\widehat{\boldsymbol{\Omega}}'-\widehat{\boldsymbol{\Omega}}\right)\psi\left(r,\widehat{\boldsymbol{\Omega}}',E,t\right)+S\left(r,\widehat{\boldsymbol{\Omega}},E,t\right).$$



Particle travel on unit sphere, $\widehat{\boldsymbol{\Omega}}$ denotes the direction of particle travel. The azimuthal direction is denoted by ω, and the polar direction by θ.

Deterministic solver for Boltzmann Transport Equatio developed by Dr. A.G. Buchan from Queen Mary University of London.



Buchan, A. G., Yang, L., & Atkinson, K. D. (2020). Predicting airborne coronavirus inactivation by far-UVC in populated rooms using a high-fidelity coupled radiation-CFD model. Scientific reports, 10(1), 1-7.

Buchan, A. G., Yang, L., Welch, D., Brenner, D. J., & Atkinson, K. D. (2021). Improved estimates of 222 nm far-UVC susceptibility for aerosolized human coronavirus via a validated high-fidelity coupled radiation-CFD code. Scientific Reports, 11(1), 1-9.

Conclusions

- FEM for flow problems on images
- Stabilised FE is a unified way to solve fluids across the scales (Darcy-Stokes-Turbulence)
- Matrix-free implementation can reduce memory costs and suitable for HPCs
- Massive parallel x86 CPU platform scaled well
- Algorithm development: multiphase flow, solid mechanics, fluid-structure interaction
- On-going development: documentations, tutorial, hoping release in 2021
- On-going application: wind turbine simulation, UVC disinfection modelling

Acknowledges:

Mr. L.Balazi (CEA), previous Cranfield student

Dr. J. Yang (Total)

Dr. A.G. Buchan (Queen Mary University of London)

Prof. S. Badia (Monash University)